paraoptXII

12th International Conference on Parametric Optimization and Related Topics

September 12–16, 2022
Augsburg, Germany

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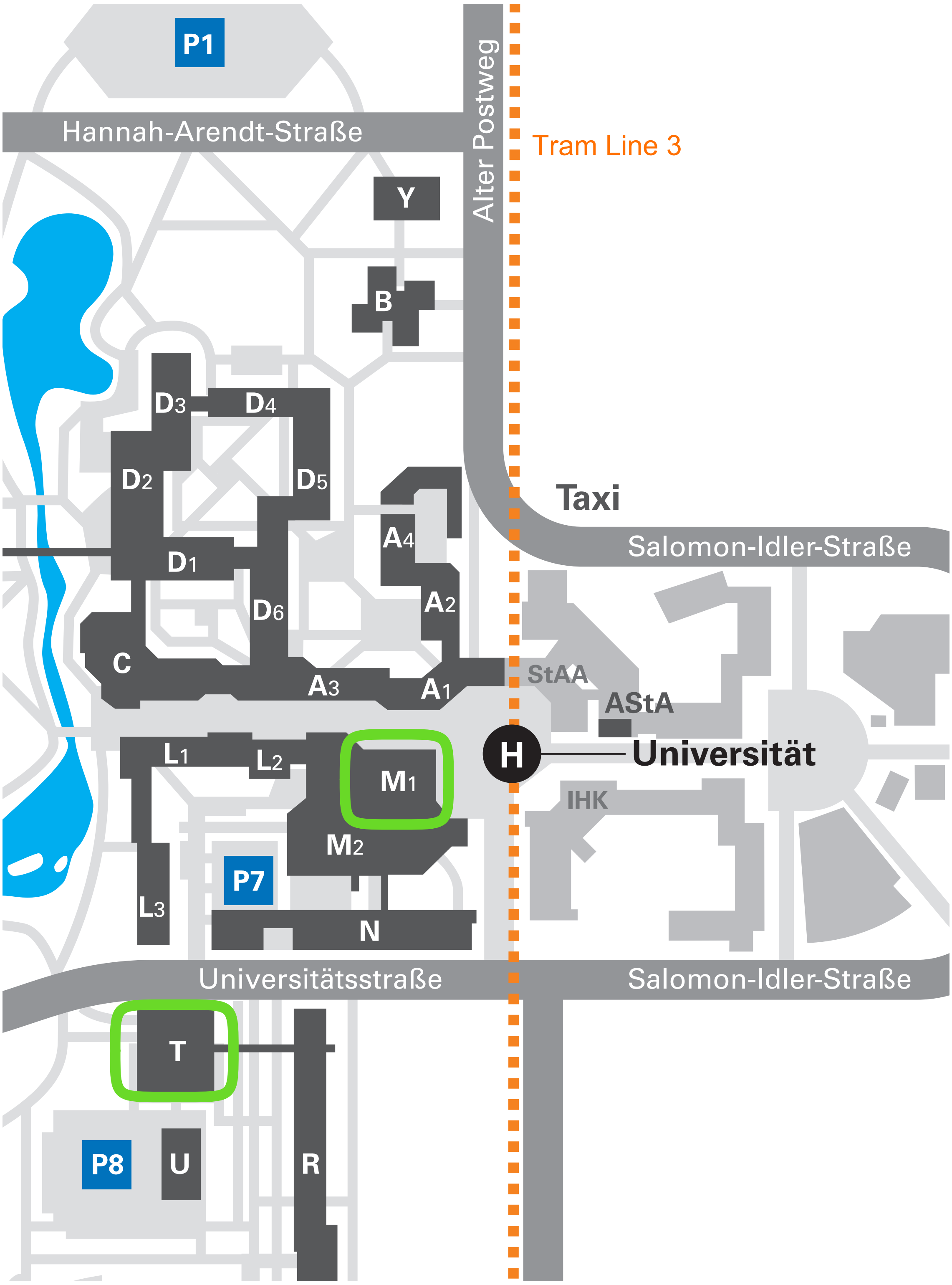
Programme and Organizing Committee:

- Mirjam Dür (University of Augsburg)
- Jan-Joachim Rückmann (University of Bergen)
- Oliver Stein (Karlsruhe Institute of Technology)
- Ralf Werner (University of Augsburg)

Universität
Augsburg
University

Map of the university campus



General Information

The conference website is: `https://uni-a.de/to/paraopt`

Conference Venue

The conference will take place in Building T at the campus of the University of Augsburg, south of the city center. From the city center of Augsburg, the university campus can be reached by tram line 3. The stop is called “Universität” (= University).

Public Transportation

Your name tag which you can collect at the registration desk is valid as a ticket on all bus and tram lines in zones 10 and 20 of Augsburg’s public transportation network during the conference week (12–16 September). Zones 10 and 20 include the trip between the university campus and most other destinations in the city of Augsburg.

Covid19

The rules governing Augsburg University will be in effect throughout the conference. These rules are being updated regularly, see
`https://www.uni-augsburg.de/en/campusleben/corona/`

Abstracts and Presentations

All abstracts are included in this booklet. All presentations are planned for 30 minutes, including time for questions. The conference language is English. A projector is available in each conference room.

Lunch (included in the conference fee)

Lunch is available in the Mensa in Building M1 on the University campus.

Note:
- It is not possible to pay by cash or credit card in the Mensa. You must use the vouchers that you can collect at the registration desk.
- The Mensa closes at 13:30.

Internet Access

Eduroam is available everywhere on the university campus.

Excursion (included in the conference fee)

On Wednesday afternoon, there will be an excursion to Lake Ammersee and the Andechs Monastery. A bus will take us to Dießen, where we will take a boat to cross the lake and go to Herrsching. From there, you can either walk up the hill to the Andechs Monastery (the walk is $\sim$4 km and has 160 meters difference in altitude) or, if you do not feel like walking, you can again take the bus to the monastery. Up there, you have some time to explore the monastery including the church where the composer Carl Orff is buried, and to visit the beer garden of the Monastery brewery (consumption there is at your own expense). In the evening, the bus will take us back to the university campus.

Departure: the bus will wait for us close to Building T and depart at 12:00.
Return: the bus will leave from the Monastery at 17:45.

Conference Dinner (included in the conference fee)

The conference Dinner will take place on Thursday at 19:00 at Riegele WirtsHaus, Frölichstraße 26, 86150 Augsburg. Before the dinner, starting at 17:30 there will be a guided tour with beer tasting through the Riegele brewery.

Program Monday 12 September 2022

All talks will take place in Room 1001 in Building T.

8:15	Registration Desk opens (in front of Room 1001 in Building T)
8:45	Welcome and Opening
9:00	Plenary talk Juan Parra: *Local and semi-local upper Lipschitz properties in linear optimization* *Chair: Jan-Joachim Rückmann*
10:00	Coffee break
10:30	Neng Fan: *Solution Path Algorithm for Distributionally Robust Linear Regression*
11:00	Jörg Fliege: *Distributed Task Assignment in a Swarm of UAVs*
11:30	Anurag Jayswal: *Robust Approach for Uncertain Multi-dimensional Fractional Control Optimization Problems* *Chair: Sonja Steffensen*
12:00	Lunch
13:30	Jesús Camacho: *Stability of the metric subregularity modulus for linear inequality systems*
14:00	Daniel Hernández Escobar: *Strongly stable stationary points for a class of generalized equations*
14:30	Harald Günzel: *A necessary condition for the stationary point setstably constituting a Lipschitz manifold* *Chair: Marco A. López*
15:00	Coffee break
15:30	Plenary talk Simone Sagratella: *Solution methods for Nash equilibrium problems with mixed-integer variables* *Chair: Oliver Stein*
16:30	Sjur Didrik Flåm: *Markets, Mathematical Programming and Prices*
17:00	Ayushi Baranwal: *Robust Absolute Value Penalty Function Method for Multi-dimensional Fractional Control Optimization Problem with Data Uncertainty* *Chair: Vladimir Shikhmann*

Program Tuesday 13 September 2022

All talks will take place in Room 1001 in Building T.

9:00	Plenary talk Amir Beck: *Sparse Regularization via Bidualization* *Chair: Mirjam Dür*
10:00	Coffee break
10:30	Ensio Suonperä: *Bilevel optimization with single-step inner methods*
11:00	Maria Carmela Ceparano: *Global convergence of affine relaxations of the best response algorithm in ratio-bounded games*
11:30	Tobias Seidel: *Solving semi-infinite optimization problems with quadratic rate of convergence: quADAPT* *Chair: Immanuel M. Bomze*
12:00	Lunch
13:30	Marco A. López: *Optimality conditions in semi-infinite convex optimization*
14:00	S.K. Gupta: *Optimality and Duality for Quasi $\epsilon-$solutions of Nonsmooth Semi-infinite Optimization Programs Involving Approximate Pseudoconvexity*
14:30	Simon Gottschalk: *An Approximation of Solution Functions in Parametric Optimization* *Chair: Jiří V. Outrata*
15:00	Coffee break
15:30	Plenary talk Francesca Maggioni: *Bounding Multistage Optimization Programs under Uncertainty* *Chair: Ralf Werner*
16:30	Antonio F. Bertachini A. Prado: *Parametric Optimization of Low Thrust Orbital Maneuvers*
17:00	Jörg Fliege: *Trajectory Optimisation of UAVs in the Presence of Adversary Sensors*
17:30	Luigi Bobbio: *Optimal Control of Autonomous Flying Vehicles in a Highly Dynamic Environment* *Chair: Neng Fan*

Program Wednesday 14 September 2022

All talks will take place in Room 1001 in Building T.

9:00	Maximilian Volk: *Weakest constraint qualifications in multi-objective optimization*
9:30	Ralf Werner: *Multiobjective robust regret*
	Chair: Stefan Schwarze
10:00	Coffee break
10:30	Immanuel M. Bomze: *Parametric copositivity – applications and open issues*
11:00	Constantin Zălinescu: *On an Open Problem Related to the Parametric Version of Gale's Example in Conic Linear Programming*
	Chair: Tuomo Valkonen
12:00	Excursion to lake Ammersee and the Andechs Monastery Lunch packages will be provided *Meeting point:* the bus will wait for us close to Building T and depart at 12:00.

Program Thursday 15 September 2022

All talks will take place in Room 1001 in Building T.

9:00	Plenary talk Nikolaus Schweizer: *Solving Maxmin Optimization Problems via Population Games* *Chair: Ralf Werner*
10:00	Coffee break
10:30	Oliver Stein: *Semi-infinite models for equilibrium selection*
11:00	Stefan Schwarze: *A branch-and-prune algorithm for discrete Nash equilibrium problems*
11:30	Vladimir Shikhman: *On local uniqueness of normalized Nash equilibria* *Chair: Patrick Mehlitz*
12:00	Lunch
13:30	Plenary talk Helmut Gfrerer: *On local analysis of multifunctions via subspaces contained in generalized derivatives* *Chair: Jan-Joachim Rückmann*
14:30	Coffee break
15:00	Jiří V. Outrata: *On the application of the SCD semismooth* Newton method to variational inequalities of the second kind*
15:30	Sonja Steffensen: *Multilevel Gas Market Model*
16:00	Daniel Hoff: *Approximation of the lower-level value function for optimistic semivectorial bilevel optimization* *Chair: Harald Günzel*
17:30	Guided tour and beer tasting at Riegele brewery (Frölichstraße 26, 86150 Augsburg)
19:00	Conference Dinner at Riegele WirtsHaus (Frölichstraße 26, 86150 Augsburg)

Program Friday 16 September 2022

All talks will take place in Room 1001 in Building T.

9:00	Plenary talk Andrea Walther: *On Nonsmooth Optimization Based on Abs-Linearization* *Chair: Oliver Stein*
10:00	Coffee break
10:30	Patrick Mehlitz: *On directional asymptotic stationarity and regularity in optimization theory*
11:00	Sebastian Lämmel: *Optimality Conditions for Mathematical Programs with Orthogonality Type Constraints*
11:30	Tuomo Valkonen: *Regularisation, optimisation, subregularity* *Chair: Mirjam Dür*
12:00	Closing Session
12:15	Lunch

Abstracts of Plenary Talks

Sparse Regularization via Bidualization

Amir Beck

Department of Statistics and Operations Research, School of Mathematical Sciences, Tel-Aviv University, Tel-Aviv, Israel. Email: becka@tauex.tau.ac.il

This talk considers the sparse envelope function, defined as the biconjugate of the sum of a squared 2-norm function and the indicator of the set of k-sparse vectors. It is shown that both function and proximal values of the sparse envelope function can be reduced into a one-dimensional search that can be efficiently performed in linear time complexity in expectation. The sparse envelope function naturally serves as a regularizer that can handle both sparsity and grouping information in inverse problems, and can also be utilized in sparse support vector machine problems.

On local analysis of multifunctions via subspaces contained in generalized derivatives

Helmut Gfrerer

Institute of Computational Mathematics
Johannes Kepler University Linz, Austria. Email: helmut.gfrerer@jku.at

It is well-known that various regularity properties of arbitrary set-valued mappings can be characterized through generalized derivatives: there are criteria for strong metric subregularity, metric regularity and strong metric regularity in terms of the graphical derivative, the limiting coderivative and the strict graphical derivative, respectively. However, the practicality of these ctiteria is very limited by the difficulty of computing the generalized derivatives, in particular the limiting coderivative and the strict graphical derivative. Thus we try to identify problem classes, which are important in applications, and where we can characterize regularity properties by means of a less complicated derivative. We propose a newly developed primal and dual derivative, whose elements are subspaces of the graphs of the strict graphical derivative and the limiting coderivative, respectively. The problem classes, for which we can apply these subspace containing derivatives (SCD), comprise the subdifferential mapping of extended real-valued functions, which are prox-regular and subdifferentially continuous on a dense subset of the graph of the subdifferential mapping. Therefore our theory is suited to analyze first-order optimality conditions for a broad class of optimization problems. We also introduce the semismooth* property for multifunctions, under which we are able to show the isolated calmness property at all points belonging to the graph of an implicitly defined multifunction and which are sufficiently close to the reference point.

Joint work with: Jiří V. Outrata

Bounding Multistage Optimization Programs under Uncertainty

Francesca Maggioni

Department of Management Information and Production Engineering, University of Bergamo, Viale Marconi n. 5 Dalmine (Italy). Email: francesca.maggioni@unibg.it

Real world decision problems are usually dynamics and affected by uncertainty. Stochastic Programming and Distributionally Robust Optimization provide two suitable modeling frameworks able to manage uncertain data in a multi-period decision making process. Hovewer, such a kind of problems are usually hard to solve since their size grows exponentially with the number of stages. For this reason approximation techniques which replace the original problem by a simpler one and provide lower and upper bounds to the optimal value are very useful in practice.
In this talk bounding methods for multistage decision problems affected by uncertainty are discussed both in stochastic and distributionally robust optimization frameworks.
In the context of multistage stochastic programs we first revise bounds from the literature based on the assumption that a sufficiently large scenario tree is given but is unsolvable and secondly when the original problem is infinite in the uncertainty dimension. In the first case, monotonic bounds based on optimal scenario grouping are discussed and compared in terms of computational complexity. In the second case, we find guaranteed bounds through the concepts of first order and convex order stochastic dominance.
In the context of multistage distributionally robust optimization, new bounding criteria are provided through scenario grouping using the ambiguity sets associated with various commonly used ϕ-divergences and the Wasserstein distance.
The proposed approaches does not require any special problem structure such as linearity, convexity and stagewise independence.
Numerical results on a multistage mixed-integer production problem show the efficiency of the proposed approaches.

Joint work with: Georg Ch. Pflug, Güzin Bayraksan, Daniel Faccini and Ming Yang.

Local and semi-local upper Lipschitz properties in linear optimization

Juan Parra

Center of Operations Research, Miguel Hernández University of Elche, 03202 Elche (Alicante), Spain. Email: parra@umh.es

In this talk we deal with canonically perturbed linear programs (tilt perturbations of the objective function and right-hand side perturbations of the constraint system). We are concerned with quantifying the stability of the feasible set and the optimal set (argmin) mappings at a nominal (given) problem by providing point-based (exclusively in terms of the nominal problem's data) upper Lipschitz-type constants. Roughly speaking, we try to estimate to which rate new feasible/optimal solutions move away from the nominal ones. One of our goals is to do this locally (around a nominal solution) through the calmness property (metric subregularity of the inverse multifunction). The other goal is to follow a semi-local approach, where by semi-local we mean to study the moving away rate of the whole feasible/optimal set around the nominal problem. This is done by means of the Lipschitz upper semicontinuity property. We will also comment on a more global approach, not restricted to problems near the nominal one, which is done by means of Hoffman constants.

Solution methods for Nash equilibrium problems with mixed-integer variables

Simone Sagratella

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A multi-agent system in many practical noncooperative frameworks can be modeled as a generalized Nash equilibrium problem (GNEP). Many solution methods have been proposed in the literature to compute solutions, or equilibria, of a GNEP, and most of them are based on reformulating the GNEP as a suitable variational inequality. Unfortunately, these reformulations are not relevant if the agents' optimization problems comprise some discrete variables. Indeed, to solve the GNEP in this mixed-integer setting, it is necessary to consider alternative methods that directly refer to the definition itself of equilibrium.
With this in mind, we consider wide classes of GNEPs with mixed-integer variables that can be provably solved by using suitable best-response methods. Moreover, we define some branch-and-bound type algorithms to solve these GNEPs with mixed-integer variables.
Most of the results presented can be found in a bunch of recently published papers, while others come from ongoing research.

Solving Maxmin Optimization Problems via Population Games

Nikolaus Schweizer

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Maxmin or minmax optimization problems arise in many applications. There is a classical connection between these problems and zero-sum games in game theory. This paper shows that, under certain conditions, maxmin solutions also arise from Nash equilibria of symmetric games, more specifically population games as studied in evolutionary game theory. Consequently, evolutionary dynamics that have been designed to find Nash equilibria in population games can be used to solve maxmin problems. We illustrate this approach in applications from operations research, quantitative finance and statistics.

Joint work with: Anne G. Balter and Johannes M. Schumacher

On Nonsmooth Optimization Based on Abs-Linearization

Andrea Walther

Institut für Mathematik, Humboldt-Universität zu Berlin . Email: Andrea.Walther@math.hu-berlin.de

For a so-called abs-smooth function, the concept of abs-linearization allows the generation of a piecewise linear local model that is provable of second order. Similar to the quadratic model generated by a truncated Taylor series in the smooth situation, this piecewise linear model can be used as a building block for optimization algorithms targeting nonsmooth problems of different kinds.
In this talk, first we define abs-smooth functions covering a wide range of applications like clustering, image restoration, and robust gas network optimization. Also various mathematical models like complementarity problems or bilevel optimization tasks can be formulated as abs-smooth functions. Subsequently, the abs-linearization approach and the properties of the resulting local model will be illustrated. Then, we discuss the solution of piecewise linear optimization problems. Based on this capability we can derive optimization approaches that allow also the handling of constraints and/or nonlinearities. For each class of the nonsmooth optimization problems and the corresponding solution approach, convergence results and numerical examples will be given.

Joint work with: Franz Bethke, Sabrina Fiege, Andreas Griewank and Timo Kreimeier

Abstracts of Contributed Talks

Robust Absolute Value Penalty Function Method for Multi-dimensional Fractional Control Optimization Problem with Data Uncertainty

Ayushi Baranwal

Department of Mathematics and Computing, Indian Institute of Technology (Indian School of Mines), Dhanbad-826004, India. Email: ayushi.19dr0025@mc.iitism.ac.in

In this paper, we use the robust absolute value penalty function method to solve a multi-dimensional first-order PDE constrained fractional control optimization problem in the face of data uncertainty. We construct an unconstrained problem by penalizing the non-fractional problem associated with the aforesaid problem using the robust absolute value penalty function and establish equivalence between them. We also show that a robust optimal solution to the considered problem is a robust minimizer of its associated penalized problem under the assumption of convex uncertain Lagrange functional. Moreover, we provide some applications to illustrate the outcomes of this paper.
To the best of our knowledge, all the results established in the paper are new in the area of multi-dimensional first-order PDE constrained fractional control optimization problem with data uncertainty.

Joint work with: Anurag Jayswal

Optimal Control of Autonomous Flying Vehicles in a Highly Dynamic Environment

Luigi Bobbio

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Southampton, Hampshire SO15 2NU. Email: l.bobbio@soton.ac.uk

Autonomous flying vehicles are becoming more and more prominent nowadays and have manifold applications, some only about to emerge: collecting traffic data, surveillance and security, disaster management, wildlife observation, delivery services, and defence. Providing such vehicles with enough computational intelligence to automatically steer according to given objectives is a challenging task, only recently tackled with control algorithms working in soft-real time and computationally feasible for the limited on-board resources. Researchers and practitioners historically address the problem concentrating on two sub-problems separately: (i) vehicle routing (VR), aiming at finding optimal itineraries for multiple vehicles visiting a set of locations, and (ii) trajectory optimisation (TO), whose purpose is to establish flight paths between locations optimising some performance measures while satisfying a set of constraints. This study aims at unifying the two aspects in a dynamic environment, also providing solution algorithms to the overall problem. First, a unique model for VR and TO is proposed where trajectory constraints are linearised through a Taylor expansion around a known solution to vehicles' equations of motion within predefined settings. Second, the above model is iteratively deployed in an adaptive configuration scheme, overcoming the local nature of Taylor's approximation and resulting in enhanced and more robust solutions. Further research developments are finally discussed.

Joint work with: Jörg Fliege and Antonio Martinez-Sykora

Parametric copositivity – applications and open issues

Immanuel M. Bomze

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Copositivity plays an important role for establishing powerful bounds of hard optimization problems, among them mixed-integer non-convex quadratic optimization under quadratic constraints and more generally, all (fractional) polynomial optimization problems. The classic situation requests searching for (dual) variables u such that a problem-specific matrix pencil value $H(u)$ is copositive with respect to a fixed (polyhedral) cone Γ, e.g. the positive orthant. This means that the matrix $H(u)$ generates a quadratic form taking no negative value over Γ. Any such u yields a rigorous bound for the original optimization problem which is tighter than the familiar Lagrangian dual bound.
The purpose of this presentation is to confront an expert audience with a situation going beyond this setting: we address the problem of finding a u such that $H(u)$ is $\Gamma(u)$-copositive where $\Gamma(u)$ is still a (polyhedral) cone which now depends in a non-linear way on u. Relevance of this question is demonstrated by two applications, (1) the dual side of the famous CDT (*aka* two-trust-region) problem, and (2) the open question to find a practical first-order condition for existence of a *homo moralis* equilibrium in a general two-person game.

Based upon joint work with: Michael Overton, Werner Schachinger and Jörgen Weibull.

Stability of the metric subregularity modulus for linear inequality systems

Jesús Camacho

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University of Elche, 03202 Elche (Alicante), Spain. Email: j.camacho@umh.es

In this talk we introduce two new variational properties, robust and continuous calmness, of the feasible set mapping for finite linear inequality systems under data perturbations, which translate into the robust and continuous metric subregularity of its inverse multifunction. The motivation of this study goes back to the seminal work by Dontchev, Lewis, and Rockafellar (2003) on the radius of metric regularity. In contrast, the unstable continuity behavior of the (always finite) metric subregularity modulus, leads us to consider the aforementioned properties. After characterizing both of them, we will derive a computable formula for the radius of robust metric subregularity and we will provide some insights on the radius of continuous metric subregularity.

Joint work with: María Josefa Cánovas, Juan Parra and Marco Antonio López

Global convergence of affine relaxations of the best response algorithm in ratio-bounded games

Maria Carmela Ceparano

Department of Economics and Statistics, University of Naples Federico II,
Naples, Italy. Email: mariacarmela.ceparano@unina.it

In a two-player non-cooperative game framework, we investigate the global convergence towards a Nash equilibrium of the affine relaxations of the best response algorithm (where a player's strategy is a best response to the other player's strategy that comes from the previous step). To be able to specify the convergence of any type of affine relaxation of the best response algorithm, we define a class of games, called ratio-bounded games, that relies on explicit assumptions on the data and that contains large classes of games broadly used in literature, both in finite- and in infinite-dimensional setting. We provide a classification of the ratio-bounded games in four subclasses such that, for each of them, the following issues are examined and answered when the strategy sets are real Hilbert spaces: existence and uniqueness of the Nash equilibria, global convergence of affine relaxations of the best response algorithm, estimation of related errors and determination of the algorithm with the highest speed of convergence.

Joint work with: Francesco Caruso (U. of Naples Federico II) and Jacqueline Morgan (U. of Naples Federico II and CSEF)

Strongly stable stationary points for a class of generalized equations

Daniel Hernández Escobar

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Bergen, Norway. Email: Daniel.Hernandez@uib.no

We consider a generalized equation that is characterized by a cone-valued mapping. It is well-known that optimality conditions for different classes of optimization problems can be formulated as such a generalized equation. In this talk, we generalize Kojima's concept of strong stability and introduce appropriate constraint qualifications. Moreover, we discuss corresponding properties between strong stability and these constraint qualifications. Finally, we apply these results to the particular class of mathematical programs with complementarity constraints and to that of mathematical programs with abstract constraints.

Joint work with: Harald Günzel and Jan-J. Rückmann

Solution Path Algorithm for Distributionally Robust Linear Regression

Neng Fan

Department of Systems and Industrial Engineering, University of Arizona,
Tucson, Arizona 85721, USA. Email: nfan@arizona.edu

In this talk, we propose a general distributionally robust regression model, which has piecewise linear loss function and elastic net penalty term. This model can generalize many other regression models. We prove the piecewise linear property of the optimal solutions to this model, which enables us to develop a solution path algorithm for the hyperparameter tuning. Doubly regularized least absolute deviations (DRLAD) regression model is proposed based this framework. A solution path algorithm is developed to speed up the tuning of two hyperparameters in this model. Numerical experiments are implemented to validate the performance of this model and the computational efficiency of the solution path algorithm.

Joint work with: Guangrui Tang

Markets, Mathematical Programming and Prices

Sjur Didrik Flåm

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Among the many institutions which valuate resources, *markets* are classical and dominant. Yet other and more modern mechanisms also help. To wit, *mathematical programming* often puts out *prices* on resources put in. That feature motivates this paper to relate programming to markets. As vehicle, it uses generalized gradients of essentially reduced objectives. As bridge, it invokes extremal convolution of such objectives. As novelties, it sheds light on bid-ask spreads, clearing prices and competitive market equilibrium.

Distributed Task Assignment in a Swarm of UAVs

Jörg Fliege

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Southampton, UK. Email: J.Fliege@soton.ac.uk

We consider the problem of distributed task assignment within a swarm of UAVs, where different UAVs can have different capabilities and need to work in teams to achieve tasks. We consider the case where communication between UAVs is costly or dangerous and should be avoided, while individual UAVs have uncertain and incomplete information at hand and new tasks can appear during mission time.

For this setting, we develop a distributed computing framework that allows for optimal task assignment under quite general conditions. At each time step of the scheme, each UAV solves a local version of an optimisation problem that encodes optimal task assignment for all UAVs. This optimisation problem takes the form of a mixed-integer linear optimisation problem that can be solved readily with state-of-the-art solvers. Our numerical results show the efficacy of this approach.

This work is a result of the Dstl-sponsored activity Maths of a Contested Electromagnetic Environment.

Joint work with: Luigi Bobbio, University of Southampton

Trajectory Optimisation of UAVs in the Presence of Adversary Sensors

Jörg Fliege

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Southampton, UK. Email: J.Fliege@soton.ac.uk

We consider a scenario in which a fleet of UAVs needs to reach a particular site of interest without being detected. The site of interest in surrounded by a network of fixed or moving sensors. Detection of a UAV is contingent on a sufficient level of detection by one or more sensors. Furthermore, assume that some UAVs are able to perform disabling/disrupting interventions to sensors along their path to the target, whereby a UAV is able to disrupt or disable all sensors within a given range for a given maximum period of time.
Our model takes explicitly into account the various measurements different sensors can take, for example received signal strength (RSS), angle-of-arrival (AOA), time-of-arrival (TOA), or time-difference-of arrival (TDOA). We provide explicit formula for probability of detection of a UAV as well as for the location error the sensors experience when trying to locate a UAV. This allows us to formulate a multiobjective mathematical programming formulation for this problem that computes optimal trajectories for all UAVs as well as optimal decisions on disabling sensors. We embed this optimisation problem into a rolling horizon optimisation approach that allows to update flight trajectories during mission time. Numerical results show the efficacy and flexibility of this approach.
This work is a result of the Dstl-sponsored activity Maths of a Contested Electromagnetic Environment.

Joint work with: Anvar Atayev, University of Warwick

An Approximation of Solution Functions in Parametric Optimization

Simon Gottschalk

Institute of Applied Mathematics and Scientific Computing, Department of Aerospace Engineering,
Universität der Bundeswehr München, 85577 Neubiberg, Germany.
Email: simon.gottschalk@unibw.de

In this talk, we present a novel approach for approximating the primal and dual parameter dependent solution functions of parametric optimization problems. For this we derive the first order necessary optimality conditions of the optimization problem for the respective parameters and reformulate them into a suitable system of equations. We replace the primal and dual solutions with approximating functions like a radial basis function or a neural network. Then, for some test parameters we find optimal coefficients as solution of a single nonlinear least-squares problem. In order to solve the least-squares problem, we apply solution strategies, which provide stationary points. We show that under mild assumptions these points are global minima and that the function approximations interpolate the solution functions at all test parameters. In addition to the construction of the approximation function, this procedure provides a cheap function evaluation criteria to estimate the approximation error. This is especially interesting for parameters, which are not part of the test parameter set. Finally, we present the numerical results of this approach applied to exemplary parametric optimization problems.

Joint work with: A. De Marchi, A. Dreves, M. Gerdts and S. Rogovs.

A necessary condition for the stationary point set stably constituting a Lipschitz manifold

Harald Günzel

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We consider parametric optimization problems with smooth problem data. The part of the stationary point set, where MFCQ holds, is known to be a topological manifold provided that the problem data come from a specific open and dense subset of the set of all possible problem data.
Now we examine this setting from the other side. We address the question under which conditions the stationary point set locally constitutes a Lipschitz manifold for all sufficiently small perturbations of the problem data.
It turns out that this, beside MFCQ, requires a specific first order condition to hold.

Joint work with: Jan Rückmann and Daniel Hernandez Escobar (both University of Bergen, Norway)

Optimality and Duality for Quasi $\epsilon-$ solutions of Nonsmooth Semi-infinite Optimization Programs Involving Approximate Pseudoconvexity

S. K. Gupta

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Roorkee-247 667, India. Email: s.gupta@ma.iitr.ac.in

The talk concentrates on investigating the results for a class of semi-infinite optimization model with cone constraints. Firstly, a necessary optimality condition for quasi $\epsilon-$ solution of the optimization model is developed using Abdaie constraint qualification. Then, the concept of quasiconvexity over cones is introduced and a sufficient optimality condition is proposed using approximate pseudoconvexity and quasiconvexity assumptions. Further, Mond-Weir and Wolfe dual problems are presented and weak, strong and converse duality results between the semi-infinite optimization model and its dual problems are proved under approximate pseudoconvexity and quasiconvexity assumptions. Moreover, to justify the results, numerical illustrations have been shown at suitable places.

Joint work with: Tamanna Yadav

Approximation of the lower-level value function for optimistic semivectorial bilevel optimization

Daniel Hoff

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In bilevel optimization one considers an optimization problem, the upper-level problem, which contains the solution set of another optimization problem, the lower-level problem, as a constraint. Within this talk, we assume that the objective function on the lower-level is vector-valued. Moreover, we follow the optimistic approach, i.e., we choose the weakly efficient points of the lower-level problem that are most suitable for the upper-level objective function. Thereby, we assume, among other, that the objective and constraint functions are sufficiently smooth and a regularity condition like the MFCQ holds at all feasible points for the lower-level problem. This allows us to ensure the upper semicontinuity of the lower-level weakly efficient set map, which is defined by the parametric multiobjective optimization problem on the lower-level. Furthermore, this assumption allows us to derive existence results for minimal and ε-minimal points for optimistic semivectorial bilevel problems. As a next step, we follow the approach of replacing the weakly efficient set of the lower-level problem by using the optimal value function and obtain the so-called lower-level value function reformulation. This provides a starting point for bounding the optimal value of the semivectorial bilevel problem. We reach this aim by bounding the value function of the lower-level by appropriate lower and upper bound sets. This serves as a base for branch-and-bound methods in optimistic semivectorial bilevel optimization.

Robust Approach for Uncertain Multi-dimensional Fractional Control Optimization Problems

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In this paper, we focus our study on a multi-dimensional fractional control optimization problem involving data uncertainty (FP) and derive the parametric robust necessary optimality conditions and its sufficiency by imposing the convexity hypotheses on the involved functionals. We also construct the parametric robust dual problem associated with the aforesaid problem (FP) and establish the weak and strong robust duality theorems. The strong robust duality theorem asserts that the duality gap is zero under the convexity notion. In addition, we formulate some examples to validate the stated conclusions.

Joint work with: Ayushi Baranwal

Optimality Conditions for Mathematical Programs with Orthogonality Type Constraints

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We consider the class of mathematical programs with orthogonality type constraints (MPOC). Orthogonality type constraints appear by reformulating the sparsity constraint via auxiliary binary variables and relaxing them afterwards. For MPOC a necessary optimality condition in terms of T-stationarity is stated. The justification of T-stationarity is threefold. First, it allows to capture the global structure of MPOC in terms of Morse theory, i. e. deformation and cell-attachment results are established. For that, nondegeneracy for the T-stationary points is introduced and shown to hold at a generic MPOC. Second, we prove that Karush-Kuhn-Tucker points of the Scholtes-type regularization converge to T-stationary points of MPOC. This is done under the MPOC-tailored linear independence constraint qualification (MPOC-LICQ), which turns out to be a generic property too. Third, we show that T-stationarity applied to the relaxation of sparsity constrained nonlinear optimization (SCNO) naturally leads to its M-stationary points. Moreover, we argue that all T-stationary points of this relaxation become degenerate and, therefore, we propose a new relaxation.

Joint work with: Vladimir Shikhman

Optimality conditions in semi-infinite convex optimization

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In this talk, different optimality conditions are presented for the semi-infinite convex optimization problem. One possible approach is to replace the set of constraints by a single constraint involving the function supremum, and appeal to different characterizations of its subdifferential. To this aim, we review two constraint qualifications which are key in these constructions. The first, called the Farkas-Minkowski property, is global in nature, while the other is a local property, which is known as the local Farkas-Minkowski property. Different Karush-Kuhn-Tucker (KKT) optimality conditions are then deduced for our optimization problem, both exact and asymptotic in nature, under progressively weaker constraint qualifications including some standard assumptions as compactness of index set, continuity of the constraints, interiority-type properties, etc.

On directional asymptotic stationarity and regularity in optimization theory

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Throughout the last years, asymptotic (or sequential) constraint qualifications, which postulate upper semicontinuity of certain set-valued mappings and provide a natural companion of asymptotic stationarity conditions, have been shown to be comparatively mild, on the one hand, while possessing inherent practical relevance from the viewpoint of numerical solution methods, on the other one.
In this talk, we aim to enrich asymptotic constraint qualifications for very general nonsmooth optimization problems over inverse images of set-valued mappings by incorporating directional data. Therefore, we first present a suitable concept of directional asymptotic stationarity which provides a necessary optimality condition even in the absence of constraint qualifications. This naturally leads us to directional versions of asymptotic constraint qualifications. We then compare these new regularity conditions with standard constraint qualifications from nonsmooth optimization. Further, we introduce directional concepts of pseudo- and quasi-normality which apply to set-valued mappings. It is shown that these properties provide sufficient conditions for the validity of directional asymptotic regularity. Finally, a novel coderivative-like variational tool is introduced which allows to study the presence of directional asymptotic regularity. For geometric constraints, it is illustrated that all appearing objects can be calculated in terms of initial problem data.

Joint work with: Matúš Benko

On the application of the SCD semismooth* Newton method to variational inequalities of the second kind

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Abstract. The first part of the lecture deals with a description of the SCD (subspace containing derivative) mappings and the SCD semismooth* Newton method for the solution of general inclusions. This method is then applied to a class of variational inequalities of the second kind. As a result, one obtains an implementable algorithm exhibiting a locally superlinear convergence. Finally we demonstrate the efficiency of a globalized version of this method via a Cournot-Nash equilibrium in which the objectives of the players (firms) are nonsmooth due to the presence of the so-called cost of change. The problem is modeled as a variational inequality of the second kind and, to test the performance of the applied method, one admits really large numbers of players and produced commodities.

Joint work with: Helmut Gfrerer and Jan Valdman

Parametric Optimization of Low Thrust Orbital Maneuvers

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The goal of the present paper is to make a numerical analysis of parametric optimization of low thrust orbital maneuver. An orbital maneuver occurs when it is necessary to modify the orbit a space vehicle to change its function or to correct effects of perturbations. A parametric optimization is made when the thrust is not free to point to any direction, but has to follow some prescribed law, like a linear or quadratic relation with time. In this case, the optimization searches for the best value of the parameters that specify the control law and the instants to start and end each burning arc. The number of arcs can be varied, but is fixed for each optimization search. It implies the use of a suboptimal control. An optimal control gives best results, in terms of minimizing the fuel consumed, but it implies in changing the direction of the thrust and the instants to turn it on and off at every instant of time, which makes the implementation of the hardware much more difficult. Besides that, the literature shows that the differences in fuel consumptions are not large and decreases when including more thrusting arcs. A point to be considered is that, when increasing the number of arcs, the duration of the maneuver increases, and perturbations like the flattening of the Earth and third-body perturbation coming from the Sun and the Moon affects the dynamics, so the fuel consumption. This is the main goal of the present paper, which extends the technique presented here to long transfer durations under this more accurate technique. The results even show that, by choosing the appropriate times and directions to apply the thrust, the fuel consumption can be lower than the equivalent ones obtained by using a dynamics without perturbations.

A branch-and-prune algorithm for discrete Nash equilibrium problems

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We present a branch-and-prune procedure for discrete Nash equilibrium problems with a convex description of each player's strategy set. The derived pruning criterion does not require player convexity, but only strict convexity of some player's objective function in a single variable. If satisfied, it prunes choices for this variable by stating activity of certain constraints. This results in a synchronous branching and pruning method. Numerical tests are performed on randomly generated instances with convex polyhedral strategy sets and convex quadratic as well as non-convex quadratic objective functions.

Joint work with: Oliver Stein

Solving semi-infinite optimization problems with quadratic rate of convergence: quADAPT

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Semi-infinite programming can be used to describe different complicated tasks in various applications. Especially in geometric tasks like the optimal usage of gemstones semi-infinite constraints can be used to describe subset and separation conditions in a general form.
Unfortunately, the simple description of the problems has its price. A semi-infinite problem is often harder to solve than a finite non-linear problem. Nevertheless different strategies have been developed to solve these problems. A classic approach, which is easy to implement, is based on discretizing the semi-infinite index set. Therefore a sequence of finite problems resulting from increasingly fine discretizations of the index sets is solved. In this talk, we show that the classical and widely used adaptive discretization algorithm of Blankenship and Falk generally leads to a slow convergence rate. Using the parametric structure of the lower level problem we propose a new way to choose discretization points adaptively. The resulting algorithm guarantees a quadratic rate of convergence under standard regularity assumptions. We present different results regarding the convergence of critical points, local and global minima. In numerical examples we compare both algorithms. In these examples our method outperforms the algorithm of Blankenship and Falk in both number of iterations and runtime.

On local uniqueness of normalized Nash equilibria

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For generalized Nash equilibrium problems (GNEP) with shared constraints we focus on the notion of normalized Nash equilibrium in the nonconvex setting. The property of nondegeneracy for normalized Nash equilibria is introduced. Nondegeneracy refers to GNEP-tailored versions of linear independence constraint qualification, strict complementarity and second-order regularity. Surprisingly enough, nondegeneracy of normalized Nash equilibrium does not prevent from degeneracies at the individual players' level. We show that generically all normalized Nash equilibria are nondegenerate. Moreover, nondegeneracy turns out to be a sufficient condition for the local uniqueness of normalized Nash equilibria. We emphasize that even in the convex setting the proposed notion of nondegeneracy differs from the sufficient condition for (global) uniqueness of normalized Nash equilibria, which is known from the literature.

Multilevel Gas Market Model

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In this talk we present a dynamic multilevel game model for the European gas market similar to a model proposed in a paper by Grimm et al [1]. The model incorporates a structure, where the technical system operator (TSO) offers certain transportation capacities that can be booked and later nominated by traders within the previously chosen bookings. The TSO defines an operational control of the gas pipeline system in order to deliver the gas according to the nominations. Moreover, following a social welfare approach he decides about booking fees to cover the transportation costs. In contrast to [1], we consider a dynamic version of the gas flow using a coupled system of isothermal Euler equations to describe the time dependent gas dynamics in the network. This leads to an optimal control problem for the TSO, which has to be coupled with the traders discrete maximization problems. Next, to the mathematical modeling of the dynamics and the coupling, we analyse the potential game structure and the necessary optimality conditions of the involved discrete generalized Nash games amongst the traders. These can further be used to aggregate the four level model into a bilevel optimal control problem. Finally, some preliminary numerical results will be given.

[1] V. Grimm, L. Schewe, M. Schmidt and G. Zöttl, A multilevel model of the European entry-exit gas market, MMOR, 2019, 89:223-255

Joint work with: Martin Gugat and Michael Schuster

Semi-infinite models for equilibrium selection

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In their seminal work ‘A General Theory of Equilibrium Selection in Games’ (The MIT Press, 1988) Harsanyi and Selten introduce the notion of payoff dominance to explain how players select some solution of a Nash equilibrium problem from a set of nonunique equilibria. We formulate this concept for generalized Nash equilibrium problems, relax payoff dominance to the more widely applicable requirement of payoff nondominatedness, and show how different characterizations of generalized Nash equilibria yield different semi-infinite optimization problems for the computation of payoff nondominated equilibria. Since all these problems violate a standard constraint qualification, we also formulate regularized versions of the optimization problems. Under additional assumptions we state a nonlinear cutting algorithm and provide numerical results for a multi-agent portfolio optimization problem.

Joint work with: Maren Beck

Bilevel optimization with single-step inner methods

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We propose a new approach to solving bilevel optimization problems, intermediate between solving full-system optimality conditions with a Newton-type approach, and treating the inner problem as an implicit function. The overall idea is to solve the full-system optimality conditions, but to precondition them to alternate between taking steps of simple conventional methods for the inner problem, the adjoint equation, and the outer problem. We prove the convergence of the approach for combinations of gradient descent and forward-backward splitting with exact and inexact solution of the adjoint equation. We demonstrate good performance on learning the regularization parameter for anisotropic total variation image denoising, and the convolution kernel for image deconvolution.

Joint work with: Tuomo Valkonen

Regularisation, optimisation, subregularity

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Regularisation theory studies the convergence of "regularised" solutions to ill-posed inverse problems as data corruption goes to zero. Similarly to the extension of first-order optimisation methods to Banach spaces, regularisation theory in Banach spaces, and non-norm-squared regularisation even in finite dimensions, generally relies upon Bregman divergences to replace norm convergence. Bregman divergences can, however, be somewhat suboptimal in terms of descriptiveness. Using the concept of (strong) metric subregularity, previously used to prove the fast local convergence of optimisation methods, we show the convergence in Banach spaces of solutions to optimisation formulations of inverse problems to a ground-truth as the data corruption vanishes. As a side result of the regularisation theory that we develop, we prove regularisation complexity results for optimisation methods: how many steps of the algorithm do we have to take for the approximate solutions to converge (in the sense of regularisation theory) as the data corruption vanishes?

This talk is based on:

[1] T. Valkonen, *Regularisation, optimisation, subregularity*, Inverse Problems 37 (2021), 045010, doi:10.1088/1361-6420/abe4aa, arXiv:2011.07575.

Weakest constraint qualifications in multi-objective optimization

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In single-objective optimization, the Guignard Constraint Qualification (GCQ) is the weakest constraint qualification guaranteeing that a local minimal point is a KKT point, i.e., that the multiplier associated with the objective function is strictly positive. In multi-objective optimization however, the GCQ is not sufficient to guarantee positive objective function multipliers at local weak, proper, and even strict Pareto optima. This fundamental difference is known as the gap between single- and multi-objective optimization. Recently, Haeser and Ramos introduced a multi-objective generalization of the regular normal cone and derived the weakest constraint qualifications which guarantee positive and strictly positive objective function multipliers at local weak Pareto optimal points. In a similar fashion, in this talk, a family of multi-objective regular normal cones is introduced and investigated. Using these generalized normal cones, the weakest constraint qualifications yielding strictly positive multipliers at local proper and strict Pareto optima are derived.

Joint work with: Oliver Stein

Multiobjective robust regret

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In this presentation, we consider multiobjective decision problems under uncertainty. In the single criteria case, robust optimization methodology can help to identify solutions which remain feasible and of good quality for all possible scenarios. An alternative method is to compare possible decisions under uncertainty against the optimal decision with the benefit of hindsight, i.e. to minimize the (possibly scaled) regret of not having chosen the optimal decision. In this exposition, we extend the concept of regret to the multiobjective setting and introduce a proper definition of multivariate (relative) regret. All early attempts in such a setting mix scalarization and modelling efforts, whereas we clearly separate both steps. Moreover, in contrast to existing approaches, we are not limited to finite uncertainty sets or interval uncertainty and further, computations remain tractable in important special cases. In the talk we will pay special attention to the numerical solution of the multiobjective problem formulation.

Joint work with: Patrick Groetzner

On an Open Problem Related to the Parametric Version of Gale's Example in Conic Linear Programming

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We are concerned by the following conic linear programming problem

(P$_{c,b}$) minimize $c(x)$ s.t. $x \in P,\ Ax - b \in Q$,

and its dual

(P$^*_{c,b}$) maximize $y^*(b)$ s.t. $y^* \in Q^+$, $A^*y^* - c \in -P^+$,

where X, Y are Hausdorff locally convex spaces, X^* and Y^* are their topological dual spaces, $A : X \to Y$ is a continuous linear operator, $A^* : Y^* \to X^*$ is the adjoint of A, $P \subset X$ and $Q \subset Y$ are convex cones, $P^+ \subset X^*$ and $Q^+ \subset Y^*$ are the positive dual cones of P and Q, $b \in Y$ and $c \in X^*$.

Our first aim is to give an answer to the open problem concerning the perturbed Gale's example considered in [1,p. 12]. The second aim is to derive the main duality results for problems (P$_{c,b}$) and (P$^*_{c,b}$) using Rockafellar's perturbation method (see [2,3]), which seems to be more direct than those used in [4], [5] and [6] (which associates certain convex sets to these problems), or the one from [7] (involving the Lagrangian function associated to (P$_{c,b}$)).

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