Relational Mechanics

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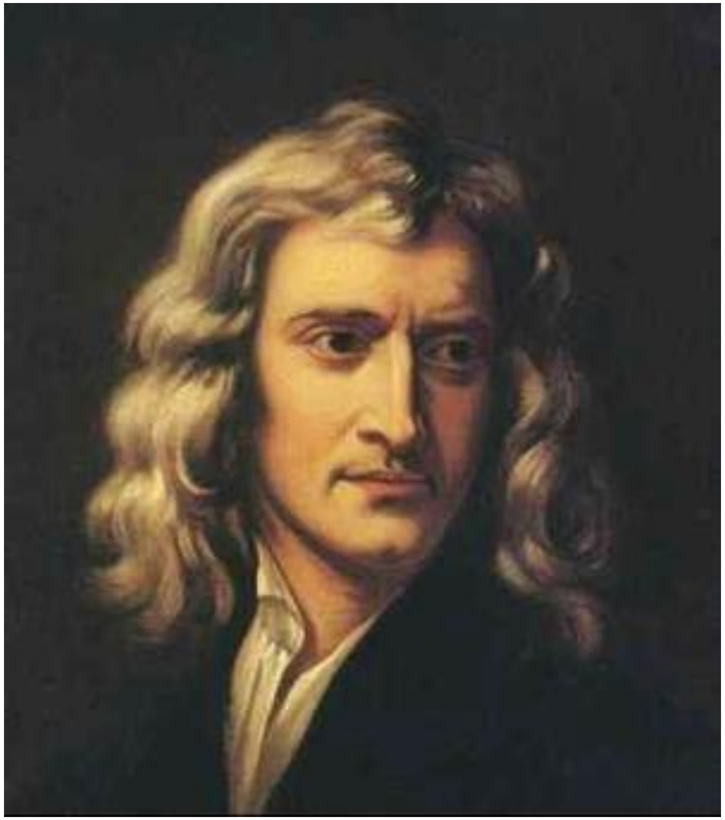
Isaac Newton (1642 – 1727)

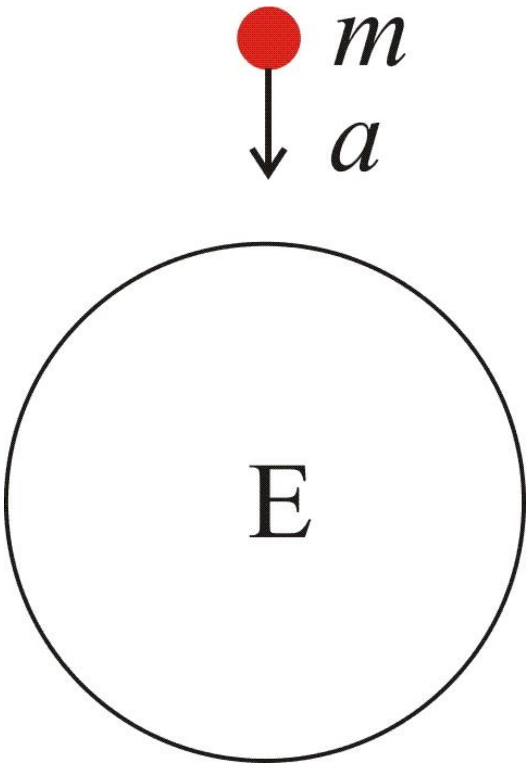


1687: Principia

$$F = ma$$

$$F = G\frac{m_1 m_2}{r^2}$$

Free fall in Newtonian Mechanics

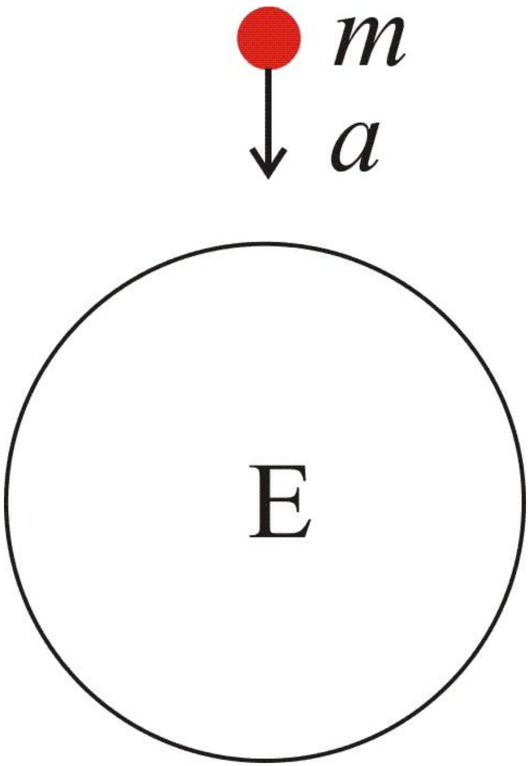


$$F = ma$$

$$mg = ma$$

$$a = g = \frac{GM}{R^2} = 9.8\,\frac{m}{s^2}$$

Free fall in Newtonian Mechanics



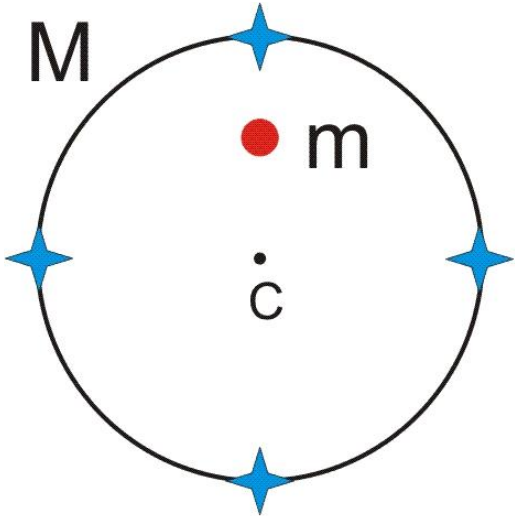
$$F = ma$$

$$mg = ma$$

$$a = g = \frac{GM}{R^2} = 9.8\,\frac{m}{s^2}$$

However, this is <u>not</u> a two-body problem. There are also stars and galaxies around the Earth.

$$F = G\frac{m_1 m_2}{r^2}$$



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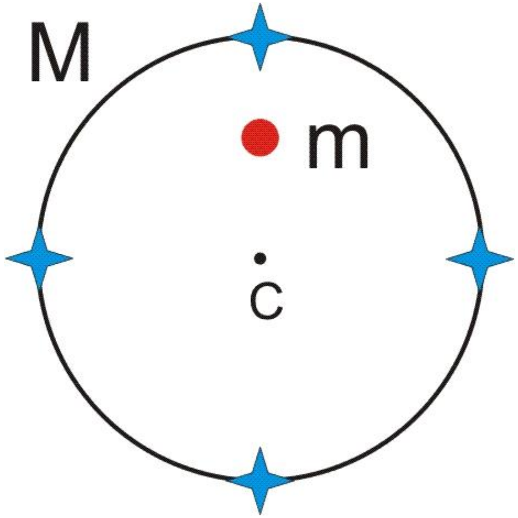
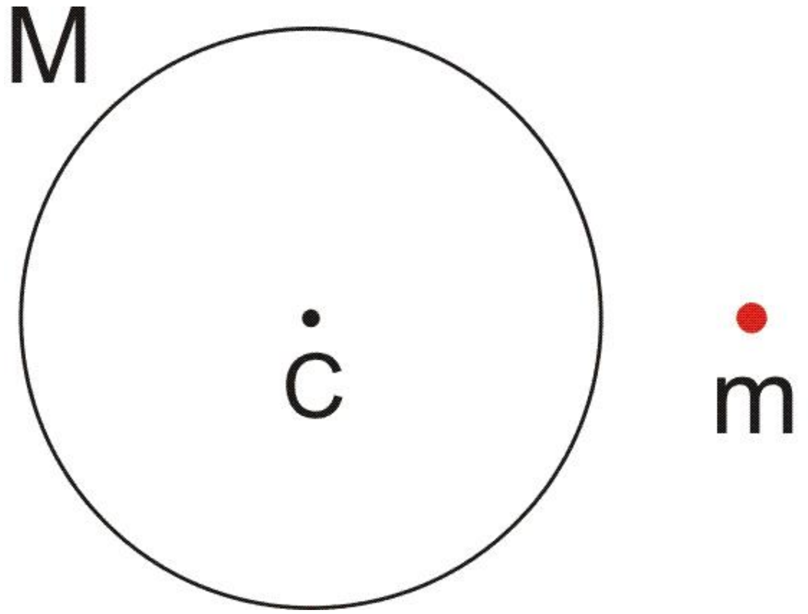


Newton in the Principia

"Theorem 30: If to every point of a spherical surface there tend equal centripetal forces decreasing as the square of the distances from these points, I say, that a corpuscle placed within that surface will not be attracted by these forces any way."

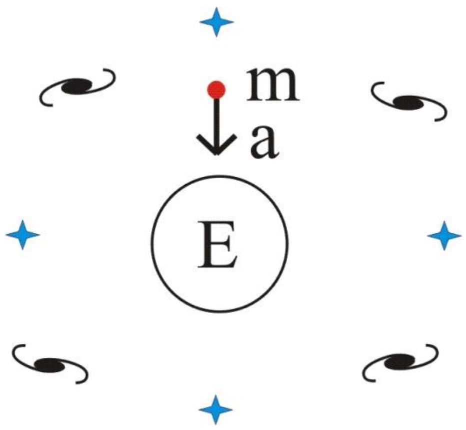
$$F = 0$$

"Theorem 31: The same things supposed as above, I say, that a corpuscle placed without the spherical surface is attracted towards the centre of the sphere with a force inversely proportional to the square of its distance from that centre."



$$F = G\frac{M_g m_g}{r^2}$$

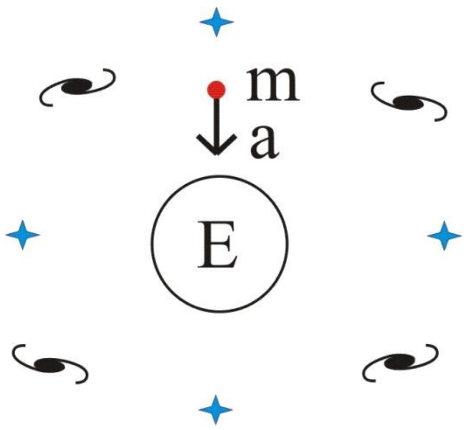
Free fall in Newtonian Mechanics:



$$F_E + F_* = m_i a$$

$$G\frac{m_g M_{gE}}{R_E^2} + 0 = m_i a \quad \Rightarrow \quad a = \frac{m_g}{m_i}\frac{GM_{gE}}{R_E^2}$$

Free fall in Newtonian Mechanics:

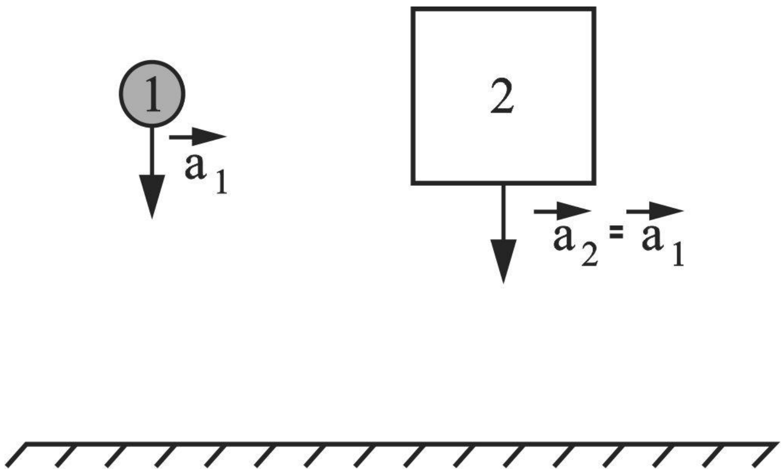


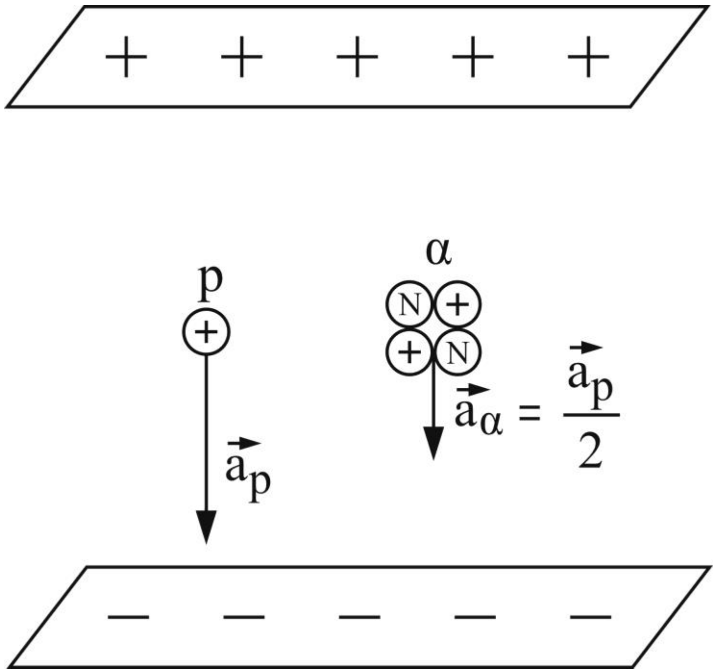
$$F_E + F_* = m_i a$$

$$G\frac{m_g M_{gE}}{R_E^2} + 0 = m_i a \quad \Longrightarrow \quad a = \frac{m_g}{m_i}\frac{GM_{gE}}{R_E^2}$$

Galileo: cork and lead fall together. Therefore:

$$\begin{cases} \dfrac{m_{g\,Cork}}{m_{i\,Cork}} = \dfrac{m_{g\,Lead}}{m_{i\,Lead}} = 1 \\ a = \dfrac{GM_{gE}}{R_E^2} = g = 9.8\dfrac{m}{s^2} \end{cases}$$





All bodies fall freely with the same acceleration in a gravitational field.

An alpha particle moves with half of the acceleration of a proton in an electric field.

Therefore, inertia seems to be connected with gravitation and not with electromagnetism.

Newton's 2nd law of motion:

$$F = m_i a$$

The free fall acceleration of 9.8 m/s^2 is the acceleration of the apple relative to what?

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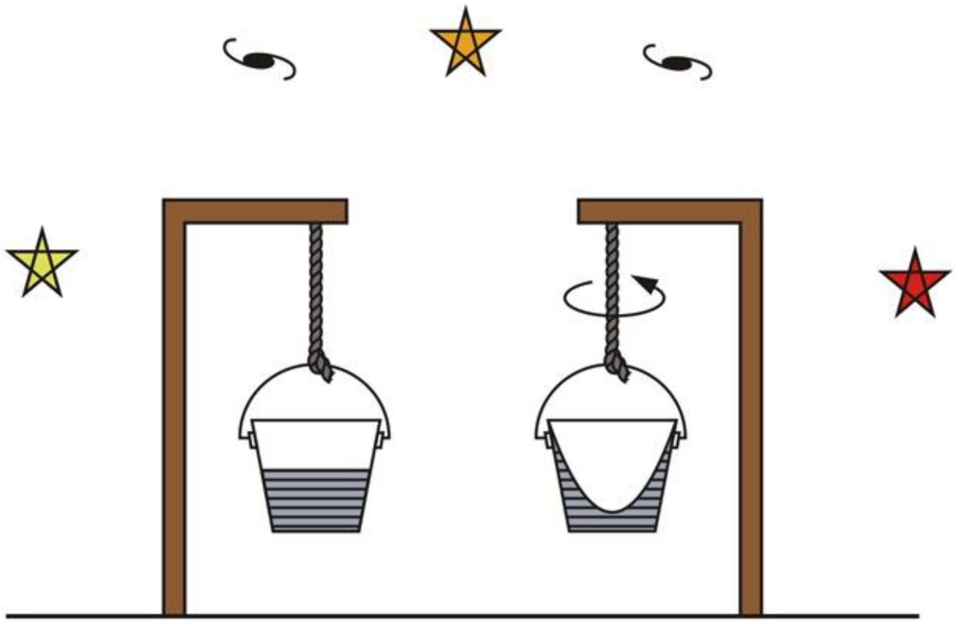
Newton in the Principia:

"Absolute space, <u>without relation to anything external</u>, remains always similar and immovable."

Newton's bucket experiment:



$$z = \frac{\omega^2}{2g} r^2$$

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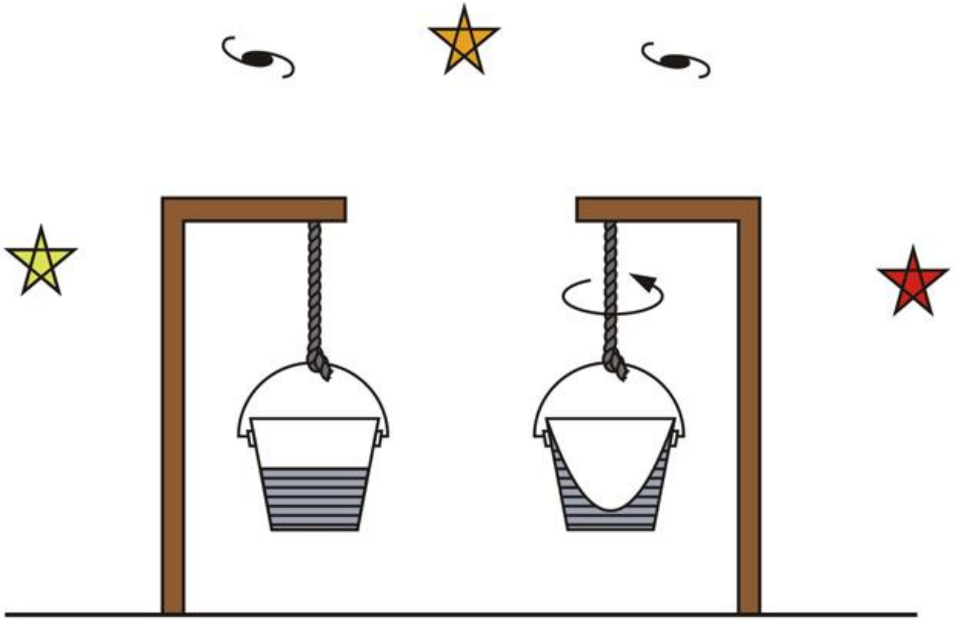


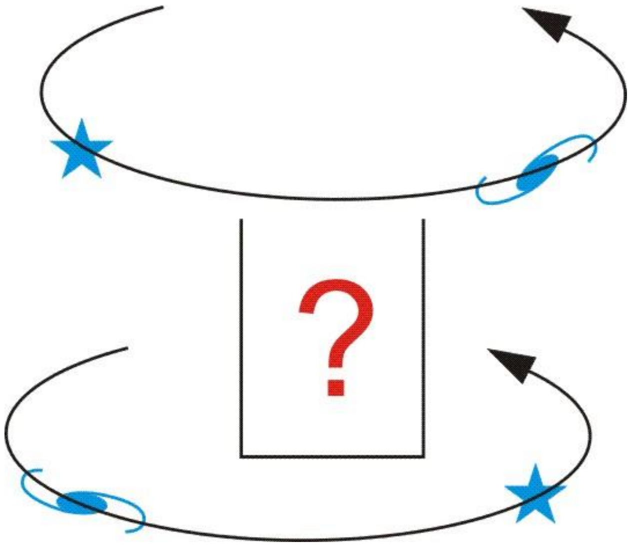
$$z = \frac{\omega^2}{2g} r^2$$

According to Newton, the concavity depends on the angular velocity ω of the water relative to absolute space, that is, relative to empty free space.

Ernst Mach in The Science of Mechanics, 1883:

- “The principles of mechanics can be so conceived, that even for relative rotations centrifugal forces arise.”
- “Try to fix Newton’s bucket and rotate the heaven of fixed stars, and then prove the absence of centrifugal forces.”

What would be the shape of the water, if it were possible to let the bucket at rest on a table, while the distant stars and galaxies rotated together once a second around the axis of the bucket?



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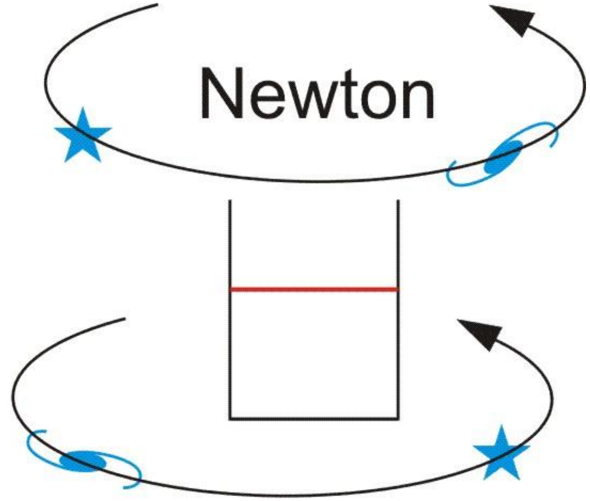

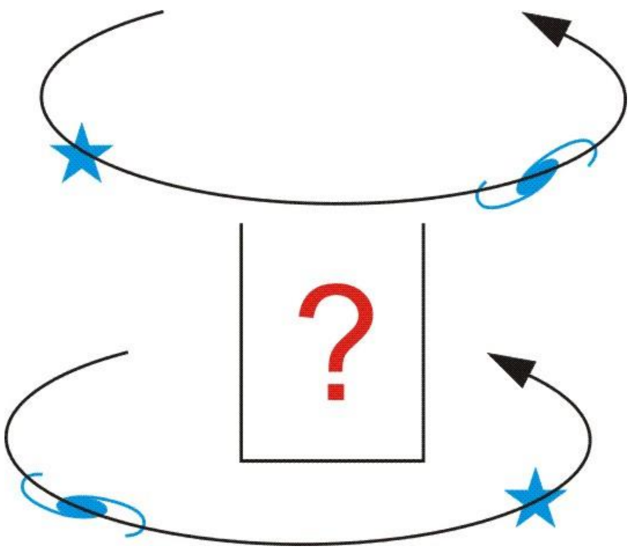

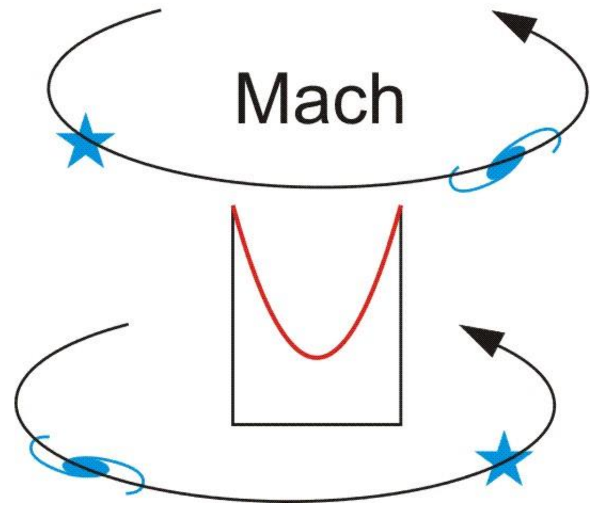

Einstein, The Meaning of Relativity, 1922:

"What is to be expected along the line of Mach's thought? A rotating hollow body must generate inside of itself a Coriolis field, and a radial centrifugal field as well."

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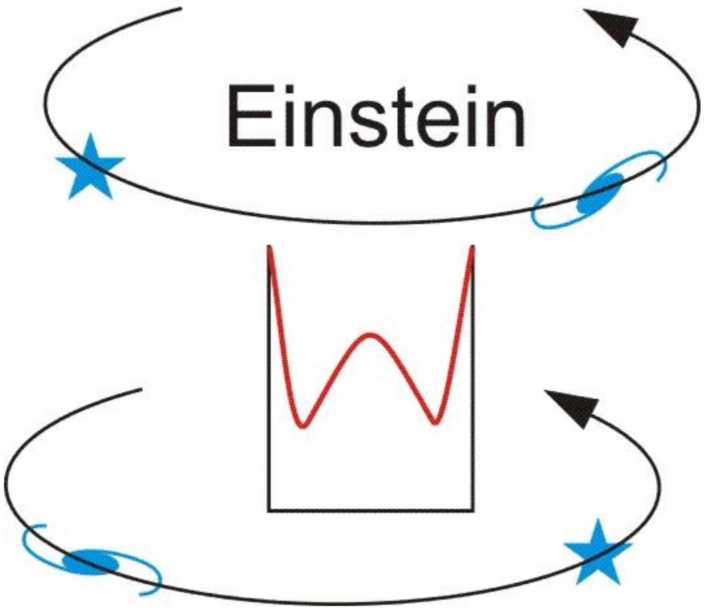
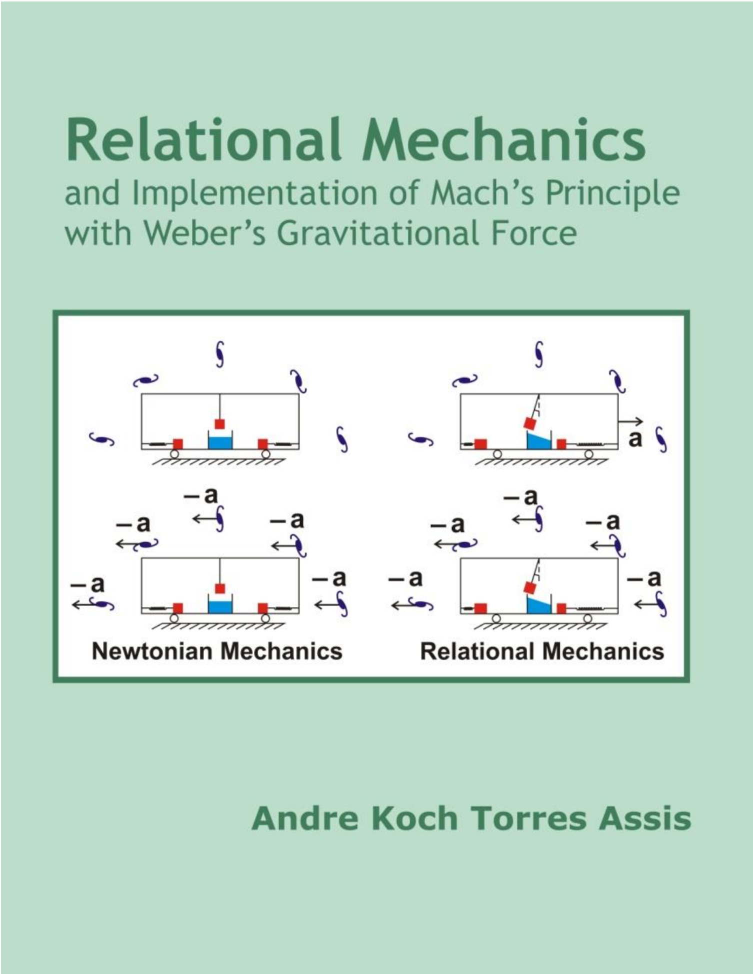
However, according to general relativity (Lense-Thirring effect):





Relational Mechanics and Implementation of Mach's Principle with Weber's Gravitational Force

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Postulates of Relational Mechanics:

The sum of all forces acting on any body is always zero in all frames of reference.

$$\vec{F} = -H_g m_{1g} m_{2g} \frac{\hat{r}}{r^2}\left(1 - 3\frac{\dot{r}^2}{c^2} + 6\frac{r\,\ddot{r}}{c^2}\right)$$

Weber (1804-1891) in 1846:

Coulomb (1785): $$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2}$$

Ampère (1822): $$\vec{F} = \frac{\mu_0}{4\pi} I_1 \, I_2 \frac{\hat{r}}{r^2} f(\alpha, \beta, \gamma)$$

Faraday (1831): $$emf = -M \frac{dI}{dt}$$

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Weber's hypothesis: $$Id\vec{\ell} \Leftrightarrow q\vec{v}$$

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left[1 + k_1 v_1 v_2 + k_2 a_{12}\right]$$

Weber's Force:

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2}\right)$$

$$\dot{r} = \frac{dr}{dt}$$

$$\ddot{r} = \frac{d^2 r}{dt^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \frac{m}{s}$$

Properties of Weber's Electrodynamics

- In the static situation ($dr/dt = 0$ and $d^2r/dt^2 = 0$) we recover the laws of Coulomb and Gauss.
- Action and reaction, conservation of linear momentum.
- Central force, conservation of angular momentum.
- It can be deduced from a velocity dependent potential energy:

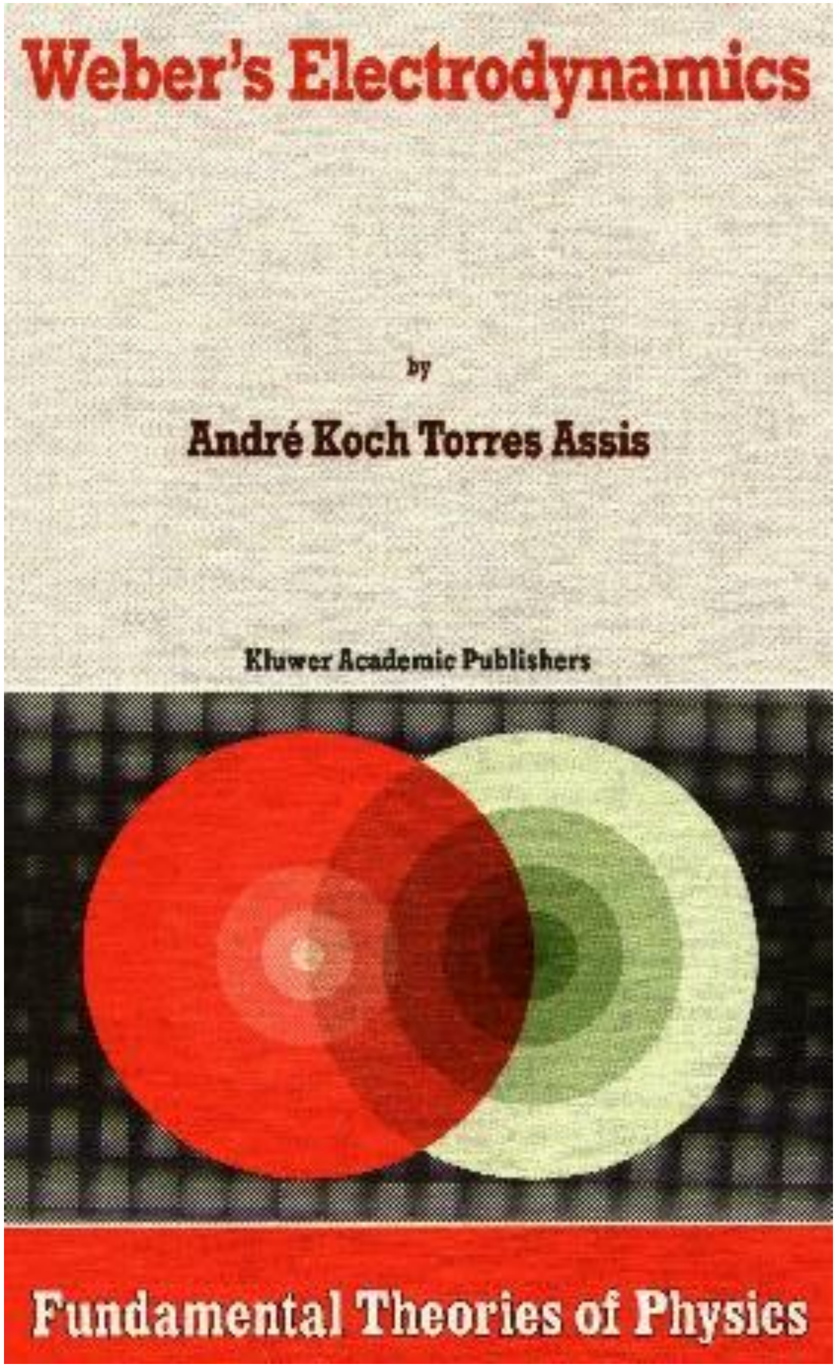
$$U = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}\left(1 - \frac{\dot{r}^2}{2c^2}\right)$$

- Conservation of energy: $$\frac{d(K+U)}{dt} = 0$$

- “Ampère’s” circuital law can be deduced from Weber’s force.

- Faraday’s law of induction can be deduced from Weber’s electrodynamics (see J. C. Maxwell, *A Treatise on Electricity and Magnetism*).

- It is completely **relational**. That is, it depends only on r, dr/dt and d^2r/dt^2. Therefore, it has the same value for all observers and in all frames of reference. It depends only on intrinsic magnitudes of the system, that is, on the relations between the interacting bodies.

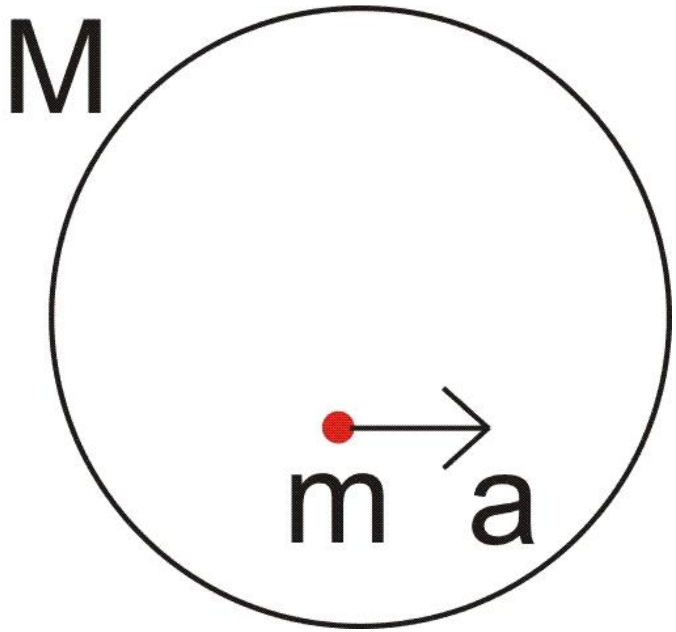


1994
Kluwer
(Springer)

Main calculation of Relational Mechanics:

$$\vec{F} = -H_g m_{1g} m_{2g} \frac{\hat{r}}{r^2}\left(1 - 3\frac{\dot{r}^2}{c^2} + 6\frac{r\,\ddot{r}}{c^2}\right)$$

Force exerted by a stationary spherical shell of mass *M* acting on a test body *m* which is moving with acceleration *a* inside it:



$$\vec{F}_{\mathrm{Newton}} = \vec{0}$$

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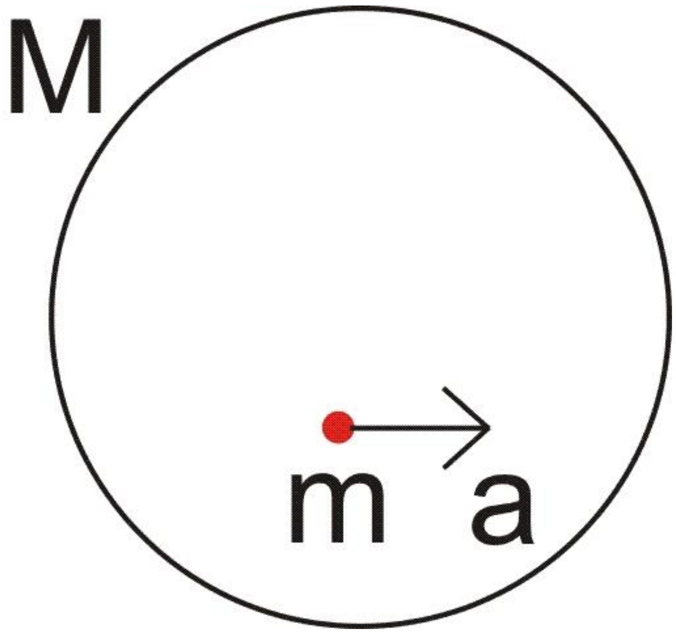


$$\vec{F}_{\text{Newton}} = \vec{0}$$

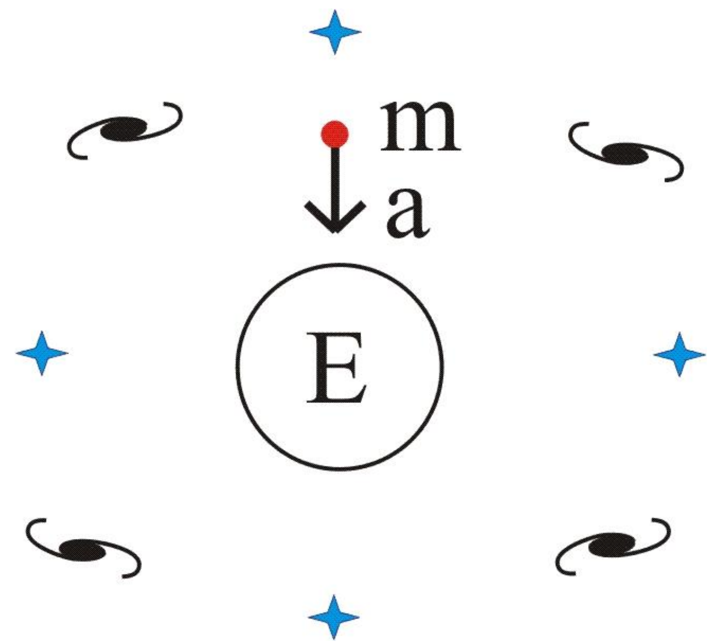
$$\vec{F}_{\text{Relat. Mech.}} = -\phi\, m_g \vec{a}$$

$$\text{with} \qquad \phi = \frac{2H_g M_g}{Rc^2}$$

Free fall in Relational Mechanics:

$$F_E + F_* = 0$$

$$H_g \frac{m_g M_{gE}}{r^2} - \Phi m_g \, a = 0$$



$$\boxed{a = \frac{H_g}{\Phi} \frac{M_{gE}}{r^2}}$$

with $$\frac{H_g}{\Phi} = \frac{H_o^2}{4\pi \, \rho_*} \approx 6.7 \times 10^{-11} \frac{Nm^2}{kg^2} = G$$

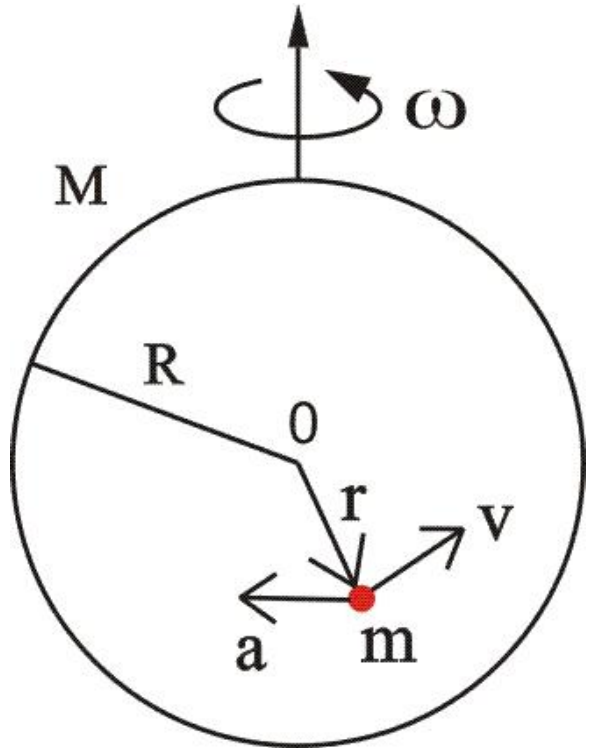
The previous slide presents the essence of Relational Mechanics.

We begin with the postulate that the sum of all forces acting on any body is always zero. Then we deduce an expression analogous to Newton's second law of motion, namely:

$$\vec{F} - m_g \vec{a}_* = \vec{0}$$

In this equation F represents the usual forces acting on the test body. The expression $-ma$ represents the gravitational force exerted by the set of distant galaxies acting on the test body, according to Weber's law applied to gravitation. The mass appearing here is the gravitational mass of the test body (that is, it is not the inertial mass, as in Newtonian mechanics). The acceleration appearing here is the acceleration of the test body relative to the frame of distant galaxies (that is, it is not the acceleration of the test body relative to absolute space, as in Newtonian mechanics).

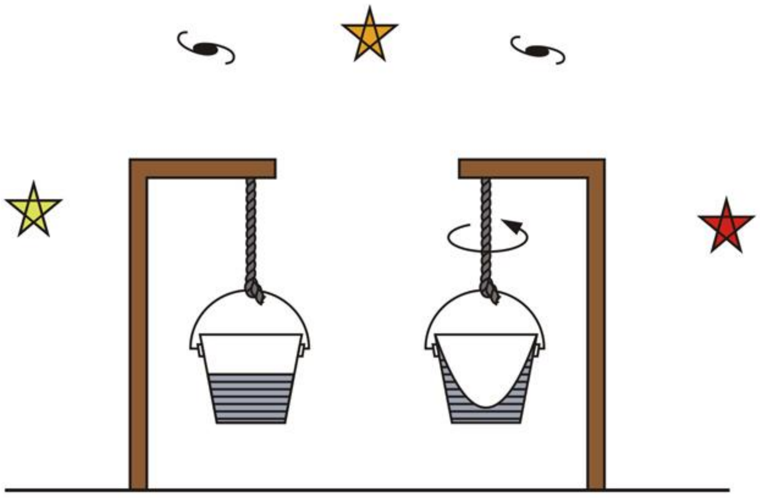
Weber's gravitational force exerted by a spinning shell acting on a particle moving inside it:



$$\vec{F} = -\frac{2H_g M_g}{Rc^2} m_g \left(\vec{a} + \vec{\omega} \times \vec{\omega} \times \vec{r} + 2\vec{v} \times \vec{\omega} \right)$$

Weber's force has a real centrifugal component and a real Coriolis component.

The bucket experiment according to Newton and Relational Mechanics:



$$z_{Newton} = \frac{\omega^2}{2g} r^2$$

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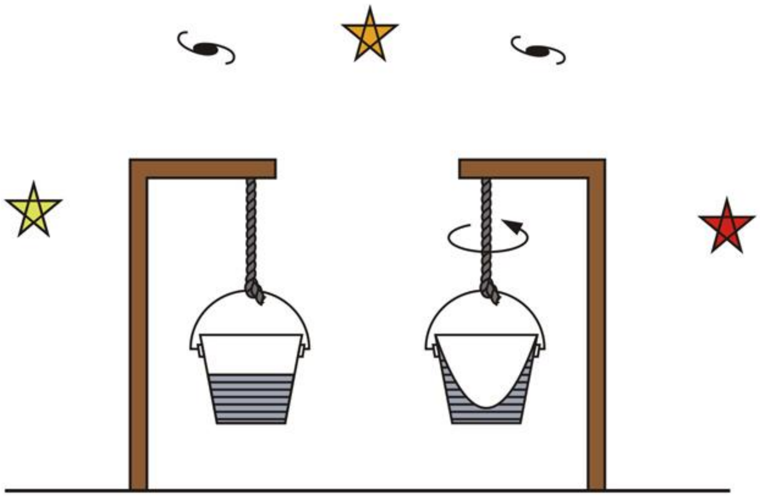
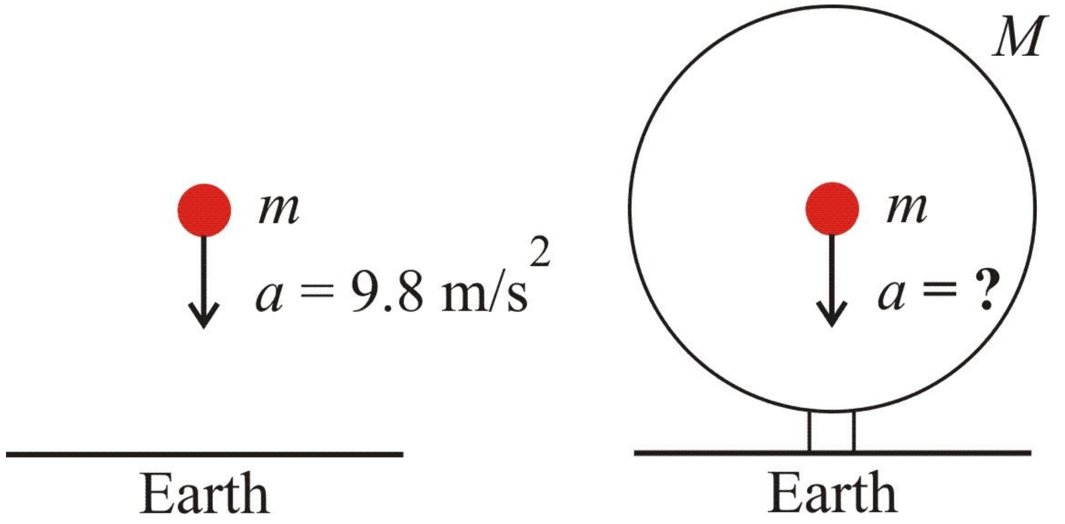


$$z_{Newton} = \frac{\omega^2}{2g} r^2$$

$$z_{\text{Relat. Mech.}} = \frac{\left(\omega_{water} - \omega_{universe}\right)^2}{2g} r^2$$

According to Relational Mechanics, the concavity depends on the <u>relative</u> angular rotation between the water and the set of distant galaxies.

Experimental test 1: What will be the acceleration of free fall when the test body is surrounded by a stationary spherical shell of mass M?



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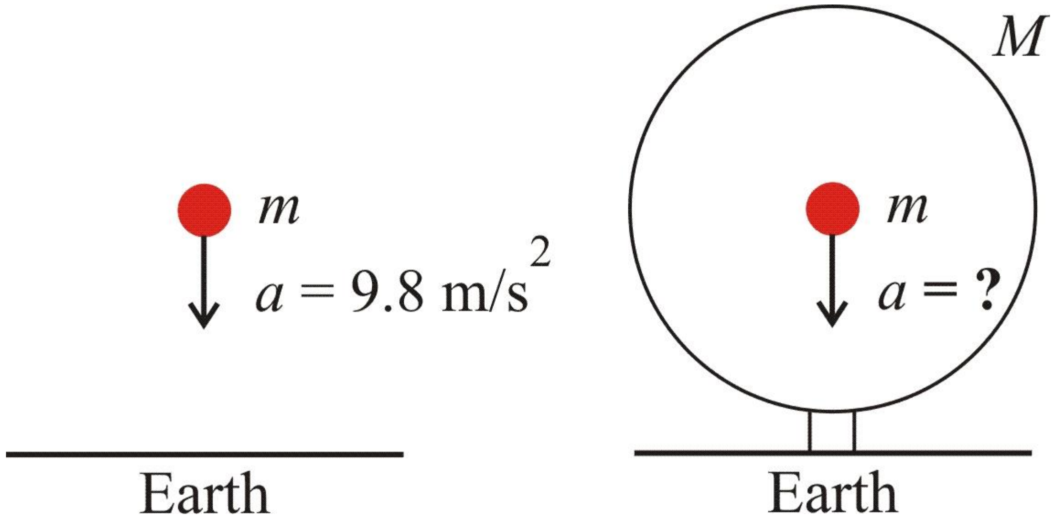


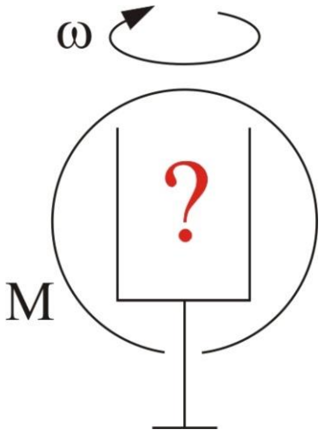
$$a_{Newton} = a_{Einstein} = g = 9.8\, m/s^2$$

$$a_{\text{Relat. Mech.}} = g\left(1 - \frac{2GM}{Rc^2}\right)$$

order of magnitude:

If $\quad R = 1m \quad$ and $\quad M = 10^3 kg \quad$ then $\quad \dfrac{2GM}{Rc^2} = 10^{-24}$

Experimental test 2: Consider a bucket at rest relative to the ground. We place a spherical shell of mass M around the bucket. What will be the shape of the water if only the shell rotates unformly around the axis of the bucket?



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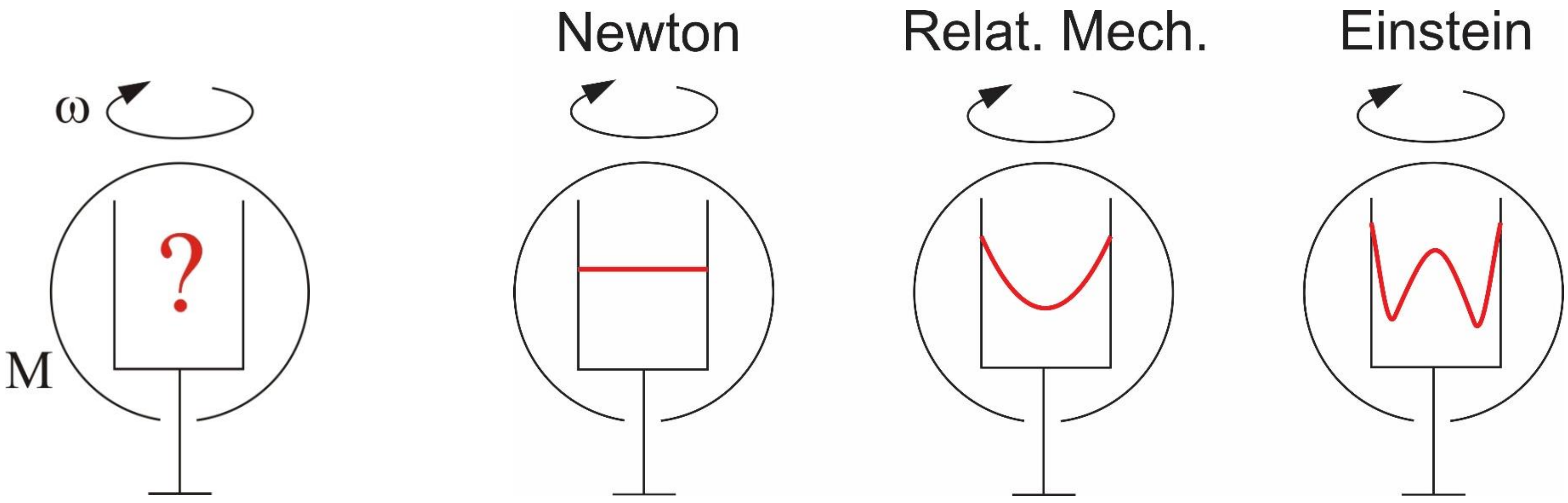

$$z_{\text{Relat. Mech.}} = k\frac{\omega^2}{2g}r^2 \qquad \text{with} \qquad k = \frac{2GM}{Rc^2}$$

Order of magnitude:

If $\quad M = 10^3\,kg,\quad R = 1\,m,\quad \omega = 1\,rad/s,\quad r = 10\,cm$

then $\quad k = 10^{-24} \qquad$ and $\qquad z = 10^{-27}\,m$

Conclusion

Postulates of Relational Mechanics:

$$\sum \vec{F} = 0$$

$$\vec{F} = -H_g m_{1g} m_{2g} \frac{\hat{r}}{r^2}\left(1 - 3\frac{\dot{r}^2}{c^2} + 6\frac{r\,\ddot{r}}{c^2}\right)$$

Main Results:

- Deduction of Newton's 2nd law: $F - m a = 0$
- Deduction of the equivalence principle: $m_i = m_g$
- We show that the centrifugal and Coriolis forces are real forces of gravitational origin exerted by the set of distant galaxies.
- Quantitative implementation of Mach's principle.

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