

Weber's Electrodynamics

Andre Koch T. Assis

University of Campinas – Brazil

www.ifi.unicamp.br/~assis



Wilhelm Weber

Wilhelm Weber (1804 – 1891)

J. C. Maxwell (1831 – 1879)

Professor of physics at Göttingen University
working in collaboration with Gauss

Weber's force (1846):

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{\hat{r}}{r^2} \left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r \ddot{r}}{c^2} \right)$$

$$\dot{r} = \frac{dr}{dt}, \quad \ddot{r} = \frac{d^2 r}{dt^2}, \quad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

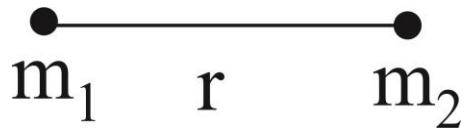
Weber measured c in 1856: $c = 3 \times 10^8$ m/s.

Therefore, connection between electromagnetism and optics before Maxwell!

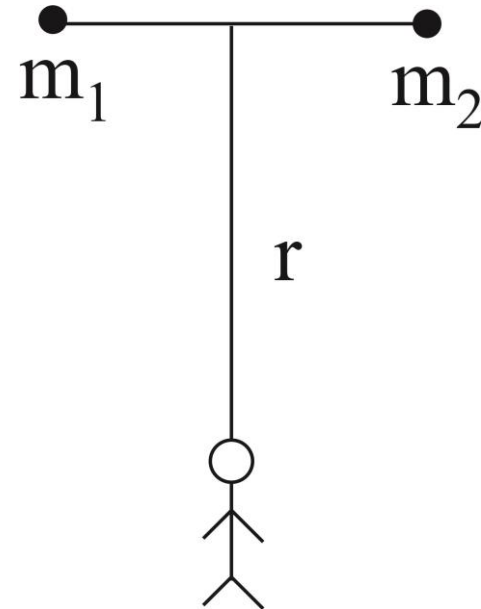
Two **different** gravitational theories, although the force law is expressed by the same equation:

$$F = G \frac{m_1 m_2}{r^2}$$

Newton's theory:



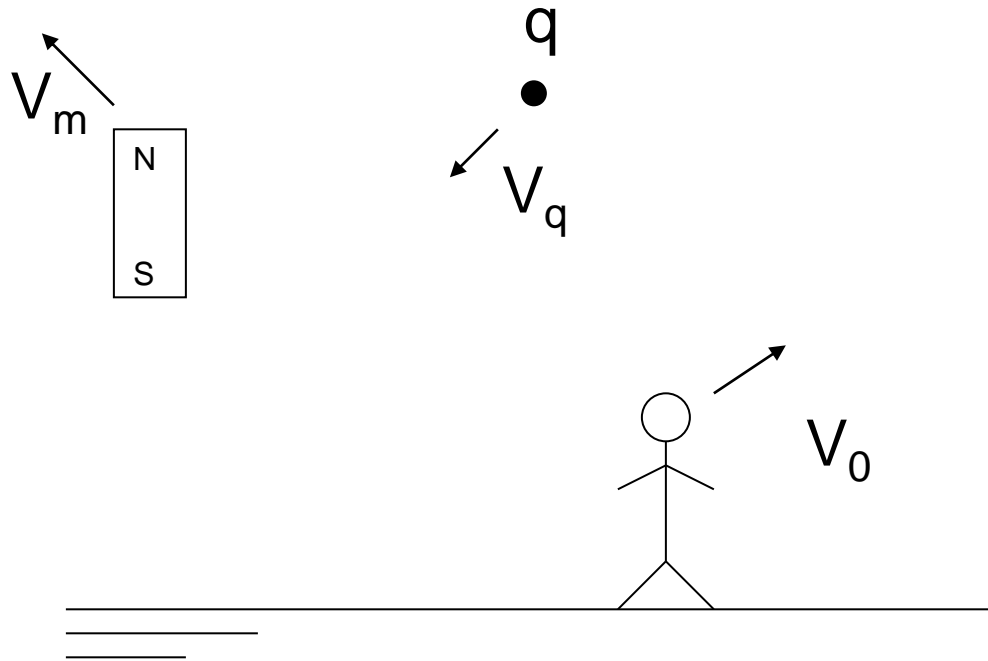
New **gravitational** theory:



Force in classical
electromagnetism:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

\vec{v} is the velocity of the charge q relative to what?



Different theories, although the force is expressed by the same equation:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

- **Maxwell (1873)**: Velocity of the charge relative to the magnetic field.
- **Thomson (1881) and Heaviside (1889)**: Velocity of the charge relative to the medium with dielectric constant ϵ and magnetic permeability μ .
- **Lorentz (1895)**: Velocity of the charge relative to the ether (reference frame of the fixed stars).
- **Einstein (1905)**: Velocity of the charge relative to the observer or frame of reference.

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

- Every textbook calls this expression Lorentz' force.
- However, it is **wrong** to call it Lorentz' force, if we interpret the velocity **v** as being the velocity of the test charge relative to the observer or frame of reference!

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{\hat{r}}{r^2} \left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r \ddot{r}}{c^2} \right)$$

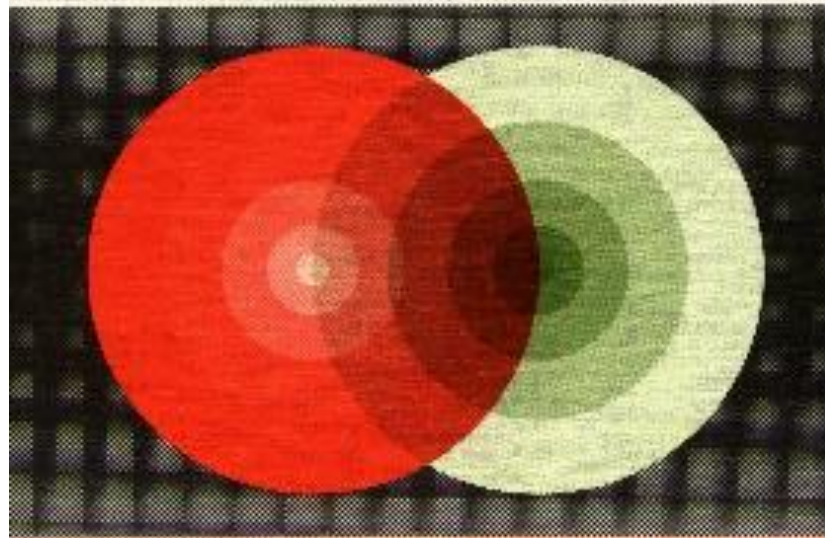
Weber's force is completely **relational**. It depends only on the distance, velocity and acceleration between the interacting charges, that is, r , dr/dt and d^2r/dt^2 . It has the same value for all observers and in all systems of reference. It depends only on magnitudes intrinsic to the system. It depends only on the relation between the bodies.

Weber's Electrodynamics

by

André Koch Torres Assis

Kluwer Academic Publishers



Fundamental Theories of Physics

1994
Kluwer
(Springer)

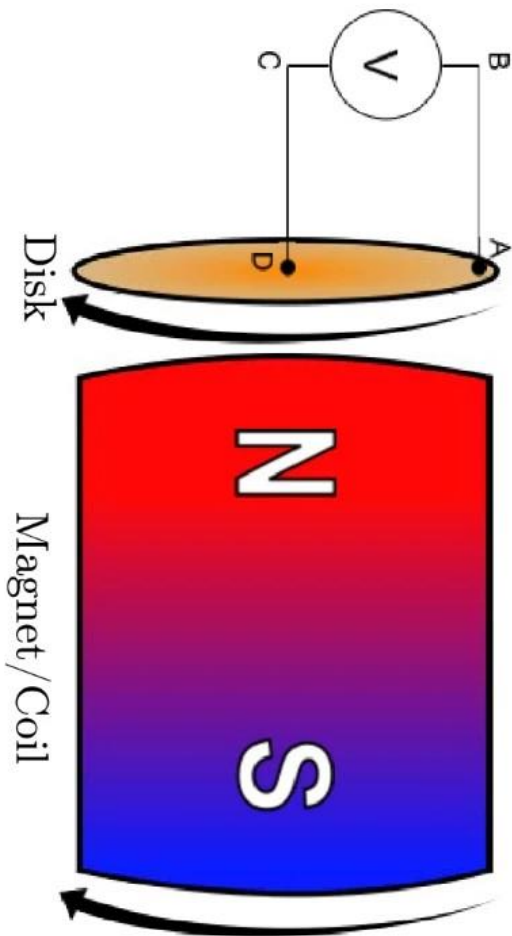
The Lagrangian and Hamiltonian formulation of Weber's electrodynamics was introduced by Carl Neumann in 1868:

$$L = K - \frac{q_1 q_2}{4\pi\epsilon_0} \frac{1}{r} \left(1 + \frac{\dot{r}^2}{2c^2} \right)$$

$$H = E = K + \frac{q_1 q_2}{4\pi\epsilon_0} \frac{1}{r} \left(1 - \frac{\dot{r}^2}{2c^2} \right)$$

= constant in time

Apparent paradoxes of unipolar induction (Faraday, 1832):



Disk	Magnet	Galvanometer
w	0	I
0	$-w$	0

Disk	Magnet	Galvanometer
0	0	0
w	w	I

Prediction of new experiments published in 1994:

A. K. T. Assis and D. S. Thober, Unipolar induction and Weber's electrodynamics, Frontiers of Fundamental Physics, M. Barone and F. Selleri (eds.), (Plenum Press, New York, 1994), pp. 409-414.

	Disk	Magnet	Closing Circuit	Galvanometer
1	0	0	0	0
2	w	0	0	I
3	0	$-w$	0	0
4	0	0	$-w$	I
5	w	0	w	0
6	0	$-w$	$-w$	I
7	w	w	0	I
8	w	w	w	0

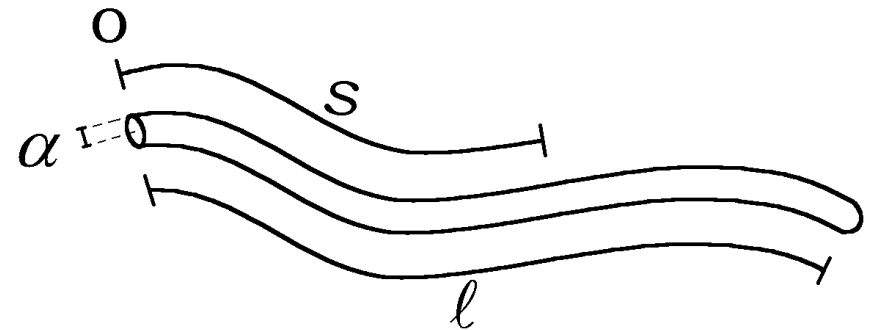
These predictions of 1994 were confirmed experimentally in 2021:

C. Baumgärtel, R. T. Smith and S. Maher, “A novel model of unipolar induction phenomena based on direct interaction between conductor charges”, Progress in Electromagnetics Research, Vol. 171, pp. 123-135 (2021).

The propagation of electromagnetic signals was first obtained by Weber and Kirchhoff in 1857 utilizing Weber's electrodynamics, before Maxwell. In particular, they obtained the telegraph equation:

$$\vec{J} = g\vec{E} = -g\left(\nabla\phi + \frac{\partial\vec{A}}{\partial t}\right)$$

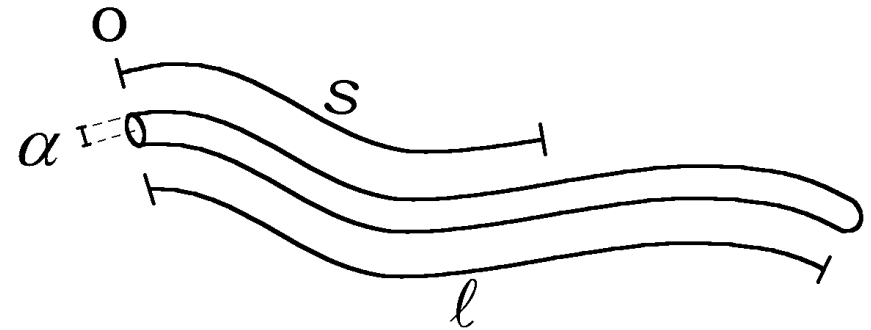
$$\nabla \cdot \vec{J} = -\frac{\partial\rho}{\partial t}$$



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$$\nabla \cdot \vec{J} = -\frac{\partial\rho}{\partial t}$$



$$\frac{\partial^2 \xi}{\partial s^2} - \frac{1}{c^2} \frac{\partial^2 \xi}{\partial t^2} = \frac{2\pi\epsilon_0 R}{\ell \ln \frac{\ell}{\alpha}} \frac{\partial \xi}{\partial t}$$

with $\xi = I, \sigma, \phi, A$

Maxwell introduced the displacement current in “Ampère’s” circuital law in 1864-1873:

$$\nabla \times \vec{B} = \mu \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

Maxwell introduced the displacement current in “Ampère’s” circuital law in 1864-1873:

$$\nabla \times \vec{B} = \mu \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

However:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Maxwell utilized the constant c which had been introduced by Weber in 1846.

$$c = 3 \times 10^8 \frac{m}{s}$$

Maxwell knew the value of this constant which had been first measured by Weber in 1856.

$$\frac{\partial^2 \xi}{\partial s^2} - \frac{1}{c^2} \frac{\partial^2 \xi}{\partial t^2} = 0$$

Maxwell knew that Weber and Kirchhoff, utilizing Weber’s force, had obtained the wave equation in 1857.

Main difference between the forces of Weber and Lorentz:

Weber's force depends on the position, velocity and acceleration \mathbf{a}_1 of the test body 1, that is, $F(r_1, r_2, v_1, v_2, \mathbf{a}_1, \mathbf{a}_2)$:

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{\hat{r}}{r^2} \left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r \ddot{r}}{c^2} \right)$$

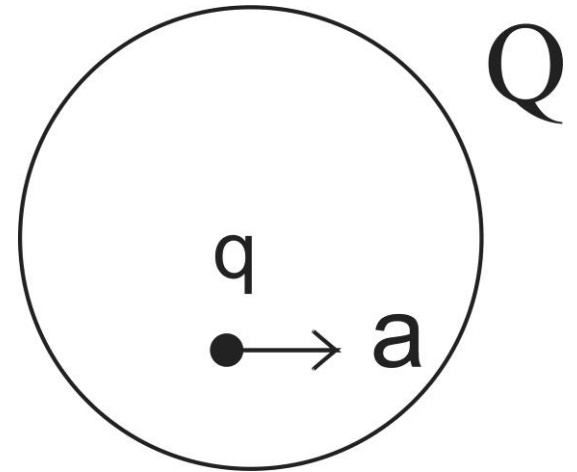
Lorentz's force depends only on the position and velocity of the test body, **but does not depend on its acceleration \mathbf{a}_1** , that is, $F(r_1, r_2, v_1, v_2, \mathbf{a}_2)$:

$$\vec{F}_{2 \text{ em } 1}^{\text{Lorentz}} = q_1 \vec{E} + q_1 \vec{v}_1 \times \vec{B}$$

Force exerted by a uniformly electrified insulating spherical shell upon an internal accelerated test body

$$\vec{F}^{Lorentz} = q\vec{E} + q\vec{v} \times \vec{B} = 0$$

$$\vec{F}^{Weber} = \frac{\mu_0 q Q}{12\pi R} \vec{a}$$

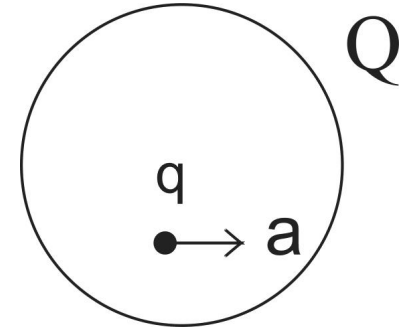


A. K. T. Assis, J. Phys. Soc. Japan, Vol. 62, pp. 1418-1422 (1993), Changing the inertial mass of a charged particle.

Equation of motion for an electron accelerated inside a uniformly charged spherical shell

Lorentz: $\vec{F} = m\vec{a}$

Weber: $\vec{F} + \frac{\mu_0 q Q}{12\pi R} \vec{a} = m\vec{a}$



or $\vec{F} = \left(m - \frac{\mu_0 q Q}{12\pi R} \right) \vec{a} = \left(m - \frac{qV}{3c^2} \right) \vec{a}$

According to Weber's electrodynamics, the electron should behave as if it had an effective inertial mass depending upon the surrounding charges:

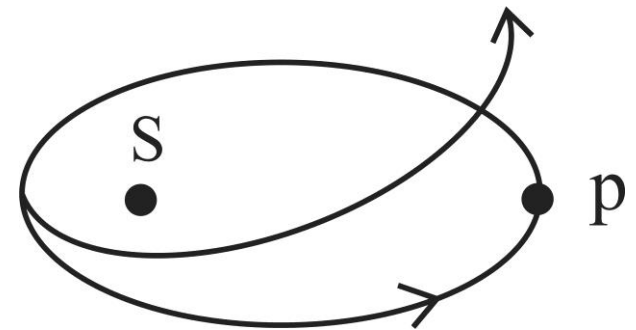
$$m_{\text{effective}} = m - \frac{qV}{3c^2}$$

This means that we can double the effective mass of an electron with a potential of 1.5 MV.

Evidence for a component of the force depending upon the acceleration of the test body:

Schrödinger derived the precession of the perihelion of the planets utilizing Weber's potential energy for gravitation, *Annalen der Physik*, V. 77, p. 325 (1925):

“The presence of the Sun has, in addition to the gravitational attraction, also the effect that the planet has a somewhat greater inertial mass radially than tangentially.”



H. Reissner in 1914 and 1915 published a potential energy for gravitation similar to Weber's potential, but he didn't quote Weber.

E. Schrödinger in 1925:
$$U = -G \frac{m_1 m_2}{r} \left(1 - 3 \frac{\dot{r}^2}{c^2} \right)$$

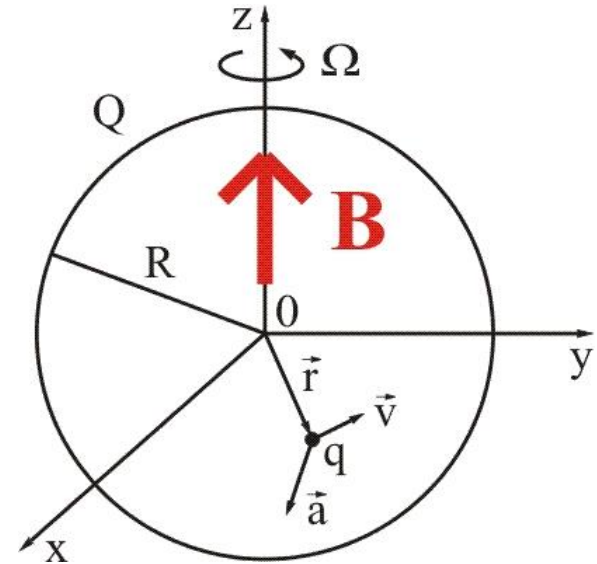
He didn't quote Weber nor Reissner.

Reissner complained to him in a letter.

The collected works of Schrödinger have been published recently. At the end of the reprint of this article there is a typewritten note, signed by Schrödinger, where he expressed apologies for Reissner for plagiarizing his ideas, unconsciously.

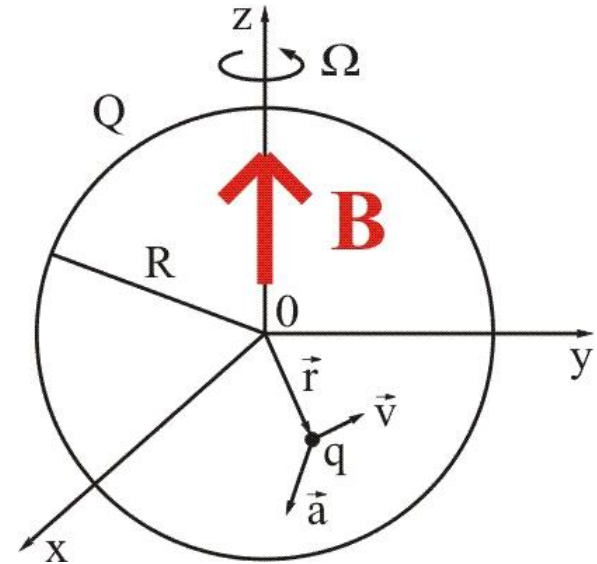
Weber had already been totally forgotten in the beginning of the XXth century!!!

Force exerted by a spinning charged spherical shell acting on an internal test charge moving relative to the shell:



$$\vec{F}^{Lorentz} = q\vec{E} + q\vec{v} \times \vec{B} = q\vec{v} \times \frac{\mu_0 Q \vec{\Omega}}{6\pi R}$$

Force exerted by a spinning charged spherical shell acting on an internal test charge moving relative to the shell:

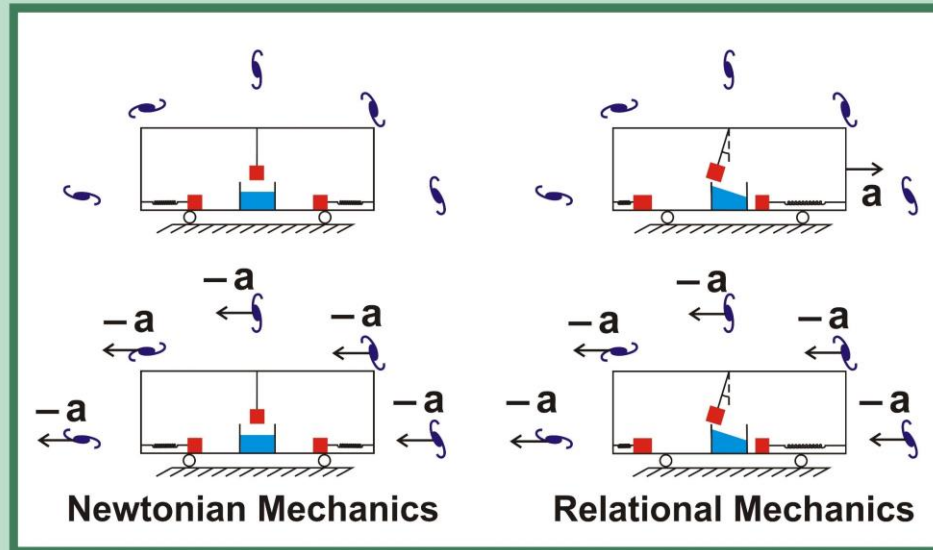


$$\vec{F}^{Lorentz} = q\vec{E} + q\vec{v} \times \vec{B} = q\vec{v} \times \frac{\mu_0 Q \vec{\Omega}}{6\pi R}$$

$$\vec{F}^{Weber} = \frac{\mu_0 q Q}{12\pi R} \left[\vec{a} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) + 2\vec{v} \times \vec{\Omega} \right]$$

Relational Mechanics

and Implementation of Mach's Principle
with Weber's Gravitational Force

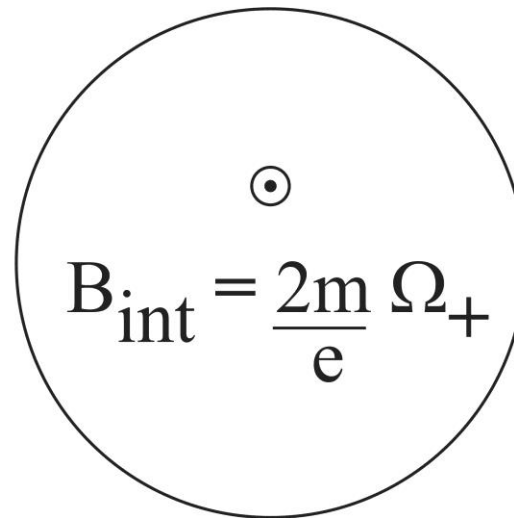
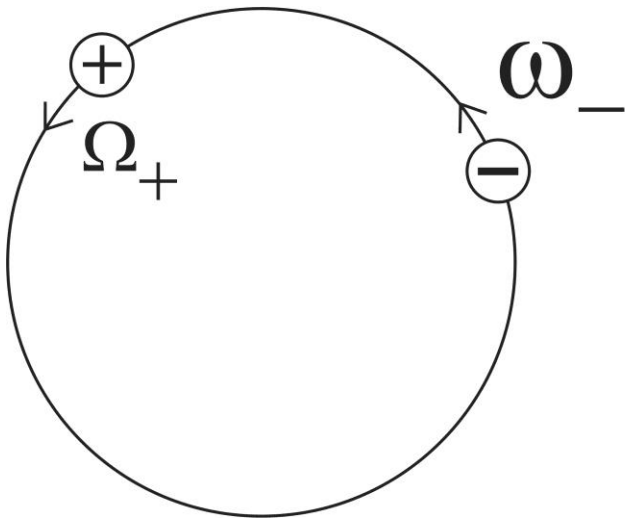


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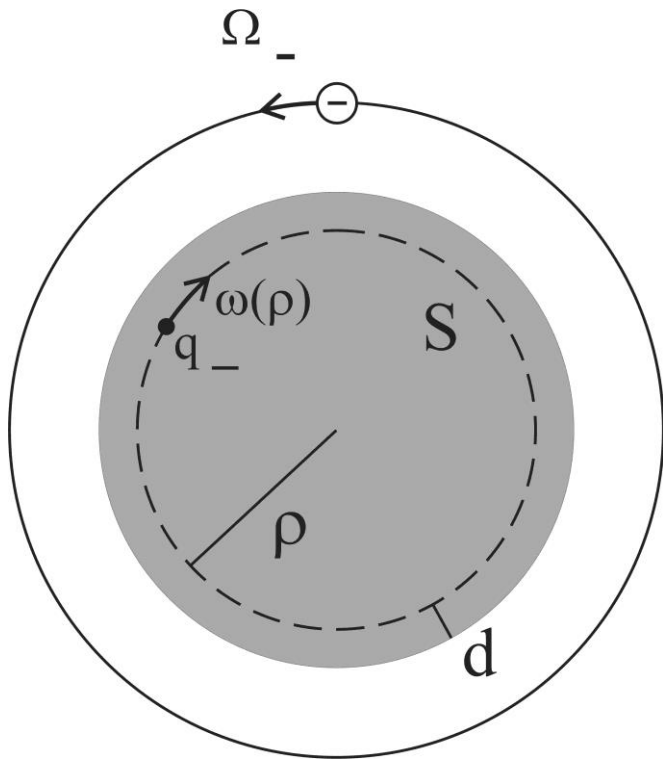
Superconductivity with Weber's Electrodynamics

$$\frac{q_1 q_2}{4\pi\epsilon_0} \frac{\hat{r}}{r^2} \left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r \ddot{r}}{c^2} \right) = m \vec{a}$$



$$B_{\text{ext}} = 0$$

London moment



$$\vec{B} = \vec{B}_{apl} e^{-d/\lambda_L}$$

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n e^2}} = \text{London penetration depth}$$

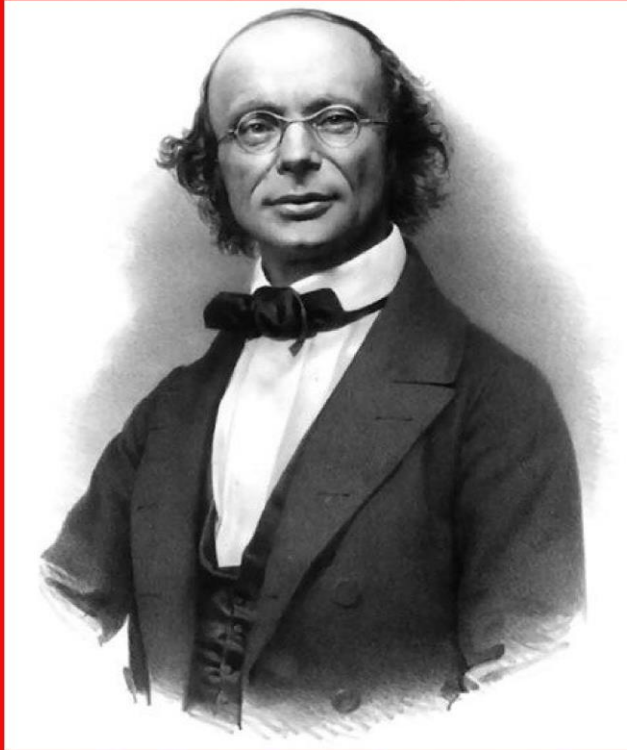
$$\vec{B}_{internal} = \vec{0} \quad \text{Meissner effect}$$

A. K. T. Assis and M. Tajmar, “Superconductivity with Weber's electrodynamics: the London moment and the Meissner effect”, *Annales de la Fondation Louis de Broglie*, Vol. 42, p. 307 (2017).

K. A. Prytz, “Meissner effect in classical physics”, *Progress in Electromagnetics Research M*, Vol. 64, p. 1 (2018).

**Wilhelm Weber's Main Works on
Electrodynamics Translated into English**

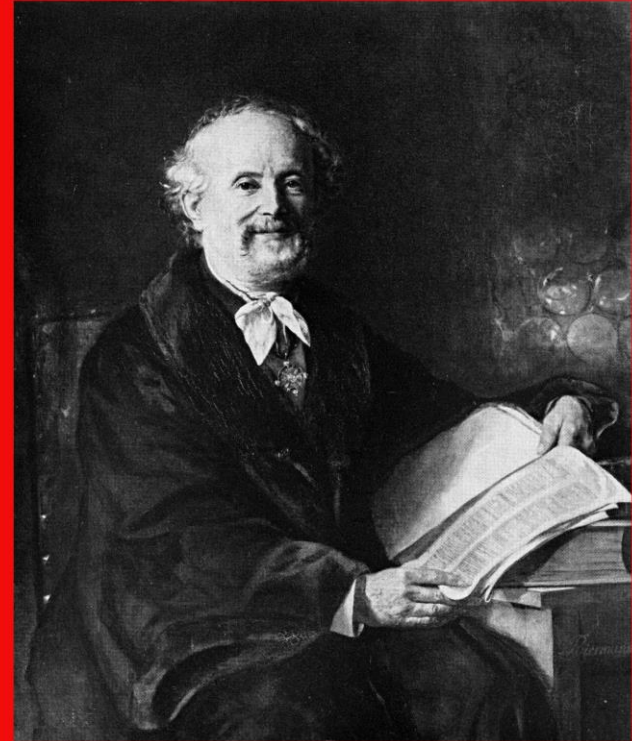
**Volume II: Weber's Fundamental Force
and the Unification of the Laws of Coulomb,
Ampère and Faraday**



Edited by Andre Koch Torres Assis

**Wilhelm Weber's Main Works on
Electrodynamics Translated into English**

**Volume IV: Conservation of Energy,
Weber's Planetary Model of the Atom and the
Unification of Electromagnetism and Gravitation**



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Conclusion

Weber's electrodynamics is extremely powerful.

In the last few years there has been a renewed interest in Weber's electrodynamics due to novel experiments and new theoretical developments.

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Extra material

Maxwell's equations (1861-1873):

Gauss's law:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

There are no magnetic
monopoles:

$$\nabla \cdot \vec{B} = 0$$

Faraday's law:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

“Ampère's” circuital law
with displacement current:

$$\nabla \times \vec{B} = \mu \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

“Lorentz's” force (1895):

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Coulomb (1785):

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{\hat{r}}{r^2}$$

Ampère (1822):

$$\vec{F} = -\frac{\mu_0}{4\pi} I_1 I_2 \frac{\hat{r}}{r^2} f(\alpha, \beta, \gamma)$$

Faraday (1831):

$$emf = -M \frac{dI}{dt}$$

Coulomb (1785):
$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{\hat{r}}{r^2}$$

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Faraday (1831):
$$emf = -M \frac{dI}{dt}$$

Weber's hypothesis:
$$Id\vec{\ell} \Leftrightarrow q\vec{v}$$

Weber's force:
$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{\hat{r}}{r^2} \left[1 + K_1 v_1 v_2 + K_2 (a_1 - a_2) \right]$$

Properties of Weber's force

- In the static case ($dr/dt = 0$ and $d^2r/dt^2 = 0$) we return to the laws of Coulomb and Gauss.
- Action and reaction: Conservation of linear momentum.
- Force along the straight line connecting the particles: Conservation of angular momentum.
- It can be deduced from a velocity dependent potential energy:

$$U = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{1}{r} \left(1 - \frac{\dot{r}^2}{2c^2} \right)$$

- Conservation of energy:
$$\frac{d(K + U)}{dt} = 0$$

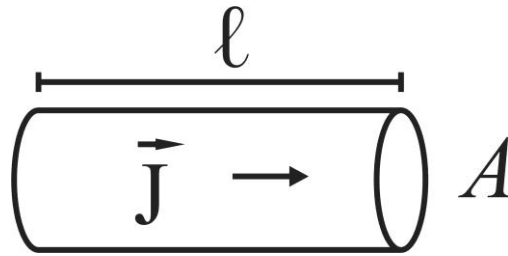
- Faraday's law of induction can be deduced from Weber's force (see Maxwell's "*Treatise on Electricity and Magnetism*", Vol. 2, Chap. 23).
- "Ampère's" circuital law can also be deduced from Weber's force.

Weber obtained the unification of the laws of Coulomb, Ampère and Faraday many years before Maxwell.

The self-inductance of a circuit may be interpreted as being due to an effective inertial mass of the conduction electrons due to their acceleration in relation to the positive lattice of the metal.

A. K. T. Assis, Circuit theory in Weber electrodynamics, Eur. J. Phys., Vol. 18, pp. 241-246 (1997).

RL circuit:



$$V = L \frac{dI}{dt} + RI$$

Newton's second law

$$F = ma = qE - bv$$

with $a = 0$

$$E\ell = \frac{b\ell}{q} v = \frac{b\ell}{q\rho A} \cdot \rho A v$$

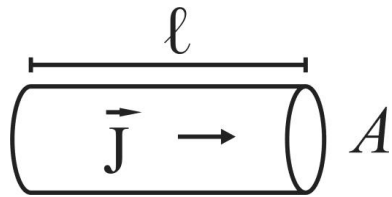
$$V = RI \quad \text{with} \quad R = \frac{b}{q\rho} \frac{\ell}{A} = r \frac{\ell}{A}$$

with $a \neq 0$

$$V = RI + \frac{m\ell}{q\rho A} \frac{dI}{dt}$$

but $\frac{m\ell}{q\rho A} \approx 10^{-16} H$

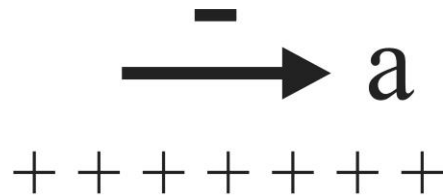
while $L = \frac{\mu_0 \ell}{2\pi} \ln \frac{2\ell}{d} \approx 10^{-6} H$



Newton's second law with Weber's force

$$F = ma = F_w - bv$$

$$\vec{F}_w = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{\hat{r}}{r^2} \left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r \ddot{r}}{c^2} \right)$$



$$F = ma = qE - bv - \left(\frac{\mu_0 q \rho d^2}{8} \ln \frac{2\ell}{d} \right) a$$

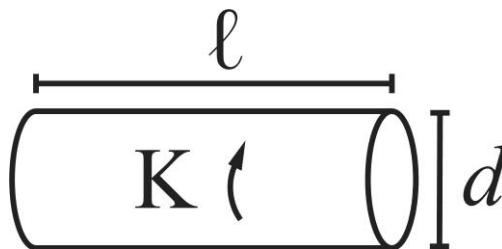
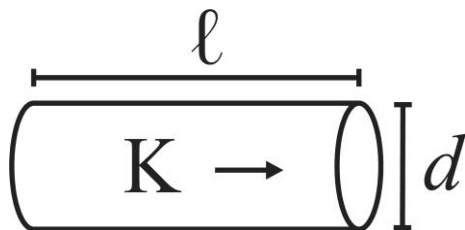
$$E\ell = \frac{b\ell}{q} v + (m_w + m) \frac{\ell}{q} a$$

where $m = 9 \times 10^{-31} \text{ kg}$ and $m_w \approx 10^{-20} \text{ kg} \gg m$

$$V = RI + L \frac{dI}{dt}$$

with

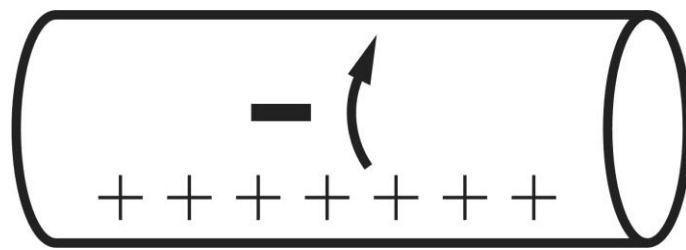
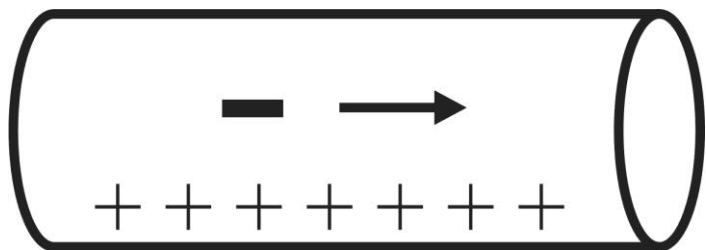
$$m_w = \frac{q \rho A}{\ell} L$$



$$V = RI + L \frac{dI}{dt}$$

$$L = \frac{\mu_0 \ell}{2\pi} \ln \frac{2\ell}{d}$$

$$L = \frac{\mu_0 \pi d^2}{4\ell}$$



$$F = ma = F_w - bv$$

$$m_w = \frac{q \rho \pi d^2}{4\ell} L_{39}$$