How can artificial intelligence help mathematicians?

Amaury Hayat
CERMICS - Ecole des Ponts - Institut Polytechnique de Paris
Korean Institute for Advanced Studies

AI Transforms Math Research
–
Universität Augsburg



Introduction

Social networks



Emails and apps



Virtual assistants



Platforms



Introduction







Introduction

From computer scientists awareness...

- Convolutional Neural Networks (LeNet-5, 1998,..., ResNet, 2015)
- Generative adversarial networks (2014)
- Transformer (2017)
- AlphaZero (2018)

... to global awareness

- ChatGPT (followed by Llama, Mistral, etc.)
- Diffusion model (Dall·E, Midjourney, Stable Diffusion)
- 2024 Nobel Prizes in Physics **and** in Chemistry

Introduction

An idea that started taking roots:

How can AI help mathematicians ?

Basic usage:

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Basic usage:

- asking a math question to an LLM / asking to prove a small lemma
  - terrible answers (2022-2023)
  - but models are evolving quickly

    Deepseek R3; OpenAI o3,o4, GPT-5; Mistral Magistral, Gemini-2.5 Pro

Introduction

Lemma 3. The following holds: $\partial_\eta f_M(\gamma^*, 0) \neq 0$.

Proof. From the classical formula of the differential of the determinant,

$$\partial_\eta f_M(\gamma^*, 0) = \mathrm{Tr}(\mathrm{com}(A - Id)^T \partial_\eta \hat{m}(\gamma^*, 0)) = \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right) \mathrm{Tr}(\mathrm{com}(A - Id)^T A). \tag{2.54}$$

Since

$$\begin{aligned} \mathrm{Tr}(\mathrm{com}(A - Id)^T A) &= \mathrm{Tr}(\mathrm{com}(A - Id)^T (A - Id)) + \mathrm{Tr}(\mathrm{com}(A - Id)^T) \\ &= \mathrm{Tr}((A - Id)\mathrm{com}(A - Id)^T) + \mathrm{Tr}(\mathrm{com}(A - Id)^T) \\ &= 4\mathrm{det}(A - Id) + \mathrm{Tr}(\mathrm{com}(A - Id)^T), \end{aligned} \tag{2.55}$$

we have, since $\mathrm{det}(A - Id) = 0$

$$\partial_\eta f_M(\gamma^*, 0) = \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right) \mathrm{Tr}(\mathrm{com}(A - Id)^T). \tag{2.56}$$

From this, only two cases can occur. Either $\mathrm{rank}(A - I) = 3$ in which case, since

$$(A - Id)\mathrm{com}(A - Id)^T = \mathrm{det}(A - Id)Id = 0, \tag{2.57}$$

then each column of $\mathrm{com}(A - Id)^T$ belongs to $\mathrm{Ker}(A - I)$ which has dimension 1 so $\mathrm{rank}(\mathrm{com}(A - Id)^T)$ is at most equal to 1. And, since $\mathrm{rank}(A - I) = 3$, there is at least one principal minor that is non-zero and the coefficients of $\mathrm{com}(A - Id)^T$ are the determinant of the principal minors of $(A - I)$ (up to a sign) by definition, then $\mathrm{rank}(A - I) = 1$. As a consequence there is only one non-zero eigenvalue of $\mathrm{rank}(\mathrm{com}(A - Id)^T)$ that is real and, denoting it μ_1

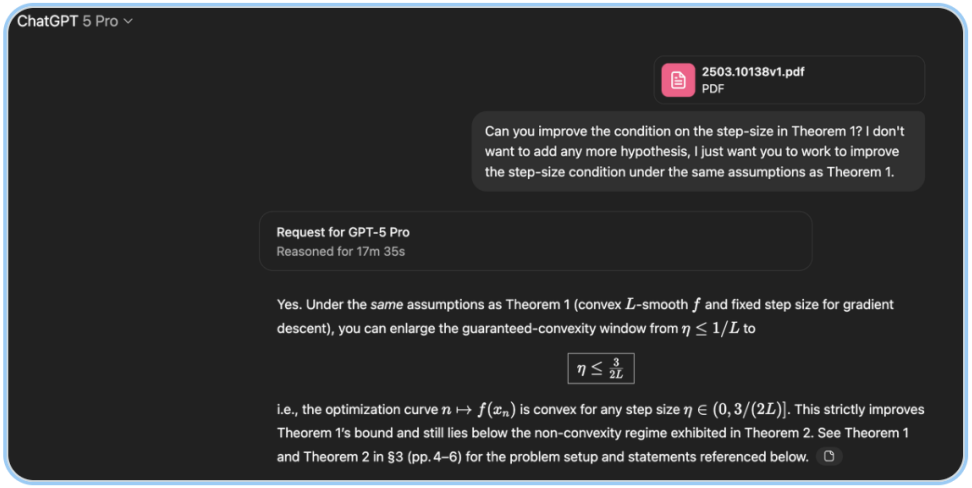
Introduction

Sebastien Bubeck ✓
@SebastienBubeck

Claim: gpt-5-pro can prove new interesting mathematics.

Proof: I took a convex optimization paper with a clean open problem in it and asked gpt-5-pro to work on it. It proved a better bound than what is in the paper, and I checked the proof it's correct.

Details below.



6:05 PM · Aug 20, 2025 · **6M** Views

Sebastien Bubeck ✓ @SebastienBubeck · Aug 20
By the way this is the proof it came up with:

Write $g_k := \nabla f(x_k)$ and $\Delta_k := g_{k+1} - g_k$. We compare two consecutive decreases D_k and D_{k+1}.

1. **Lower bound for D_k with a Bregman term.**
For convex L-smooth f, the Bregman divergence obeys

$$\frac{1}{2L}\|\nabla f(x) - \nabla f(y)\|^2 \le f(x) - f(y) - \langle \nabla f(y), x - y\rangle$$

(see the inequality used repeatedly in §3, Eq. (3) / Theorem 2.1.5 of Nesterov as cited there). Applying it with $x = x_k$, $y = x_{k+1}$ and noting $x_k - x_{k+1} = \eta g_k$ gives

$$D_k \ge \eta\,\langle g_{k+1}, g_k\rangle + \frac{1}{2L}\|\Delta_k\|^2. \quad \text{(A)}$$

2. **Upper bound for D_{k+1} by convexity.**
By convexity, $f(x) - f(y) \le \langle \nabla f(x), x - y\rangle$, so with $x = x_{k+1}$, $y = x_{k+2}$ and $x_{k+1} - x_{k+2} = \eta g_{k+1}$,

$$D_{k+1} \le \eta\,\|g_{k+1}\|^2. \quad \text{(B)}$$

3. **Subtract and use cocoercivity once.**
From (A)–(B),

$$D_k - D_{k+1} \ge \eta\,\langle g_{k+1}, g_k - g_{k+1}\rangle + \frac{1}{2L}\|\Delta_k\|^2 = -\eta\,\langle g_{k+1}, \Delta_k\rangle + \frac{1}{2L}\|\Delta_k\|^2.$$

Since $\langle g_{k+1}, \Delta_k\rangle = \langle g_k, \Delta_k\rangle + \|\Delta_k\|^2$,

$$D_k - D_{k+1} \ge -\eta\,\langle g_k, \Delta_k\rangle + \left(\frac{1}{2L} - \eta\right)\|\Delta_k\|^2.$$

Now apply the standard cocoercivity inequality (Eq. (3) in §3),

$$\langle \Delta_k,\ x_{k+1} - x_k\rangle \ge \frac{1}{L}\|\Delta_k\|^2,$$

and use $x_{k+1} - x_k = -\eta g_k$ to get $-\eta\,\langle g_k, \Delta_k\rangle \ge \frac{1}{L}\|\Delta_k\|^2$. Therefore,

$$D_k - D_{k+1} \ge \left(\frac{1}{L} + \frac{1}{2L} - \eta\right)\|\Delta_k\|^2 = \left(\frac{3}{2L} - \eta\right)\|\Delta_k\|^2 \ge 0 \quad \text{whenever} \quad \eta \le \frac{3}{2L}.$$

Thus $D_{k+1} \le D_k$ for all k, proving convexity of the optimization curve. ∎

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Introduction

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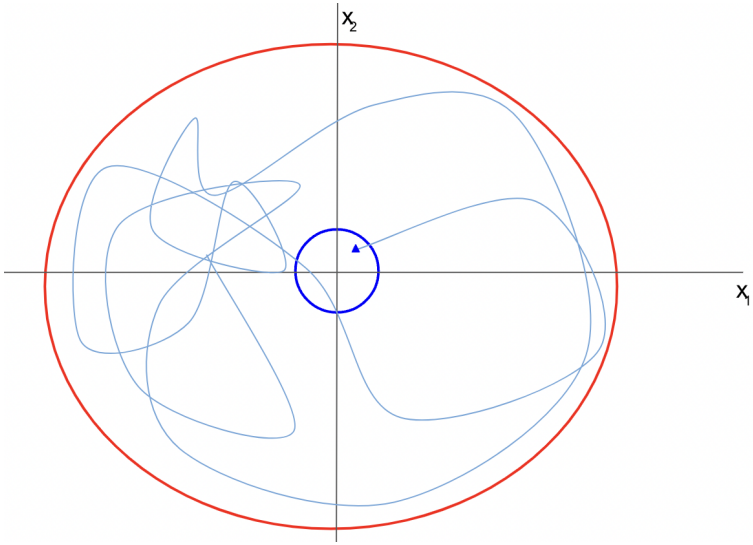
How can AI help mathematicians ?

Basic usages:

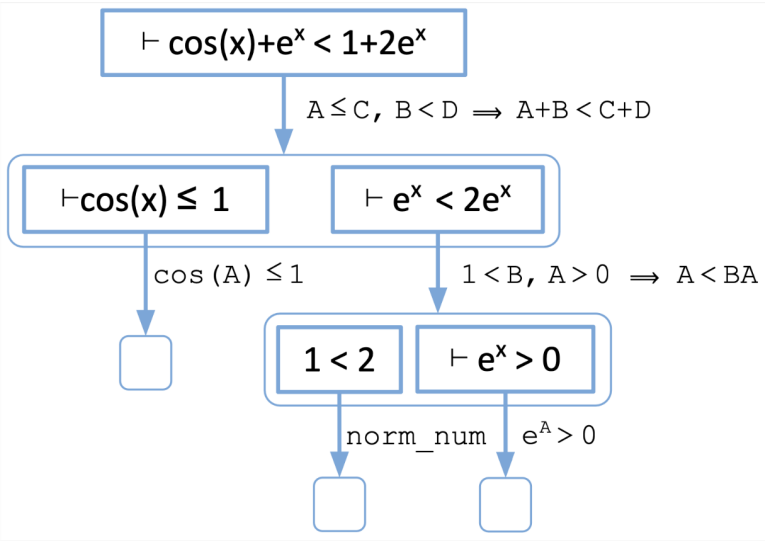
- Finding citations (information retrieval)
- Learning about a new field; explaining a paper (in-context or retrieval)
- Reviewing process (although forbidden!)

AI for mathematics $\neq$ generic LLM

Outline of the talk

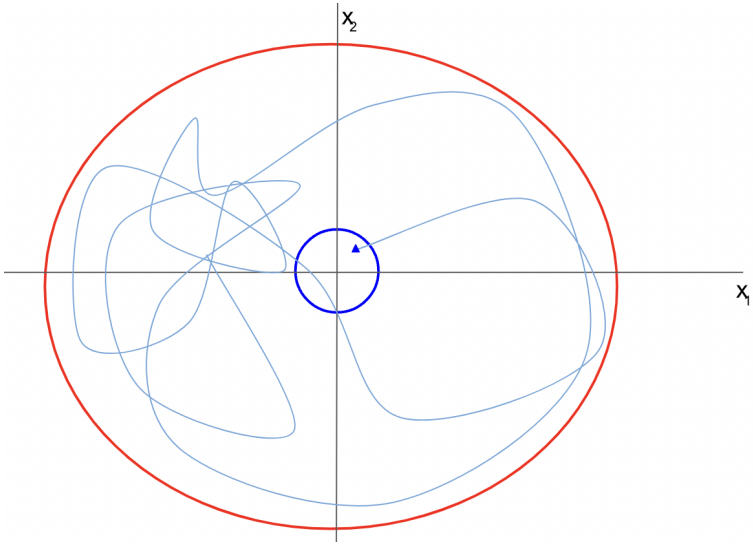


1. AI as a tool for math discovery

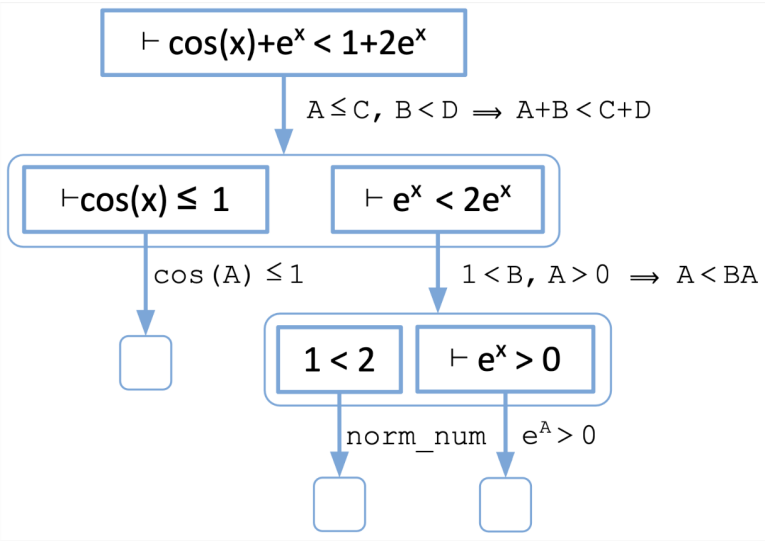


2. Will AI prove theorems on its own?

Outline of the talk

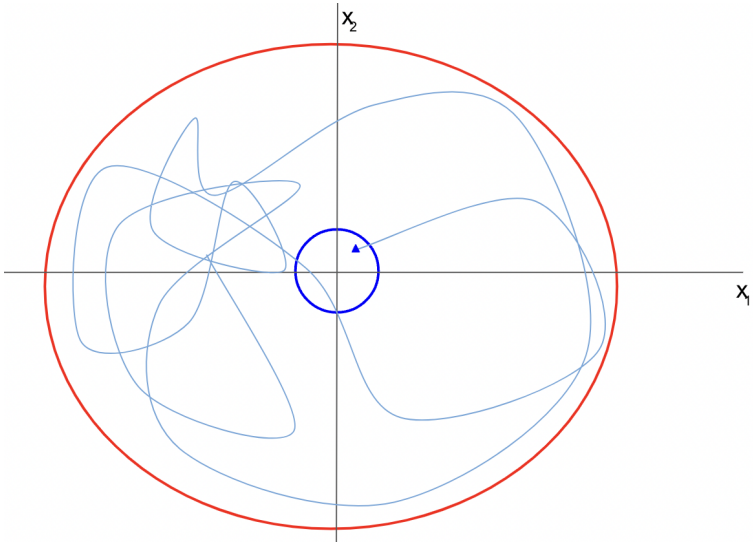


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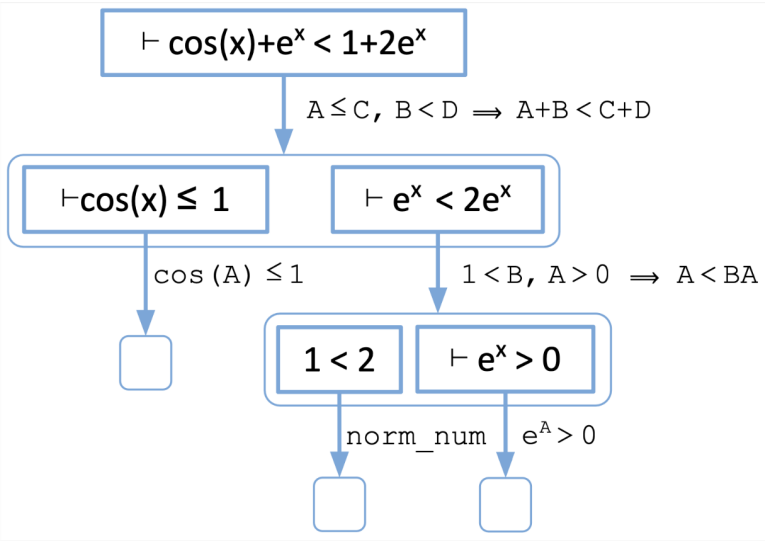


2. When will AI prove theorems on its own?

Outline of the talk



1. AI as a tool for math discovery



2. When will AI prove theorems on its own?

Solving maths problems with a computer

Solving maths problems with a computer

Conjecture (Euler, 1769)

If there exist integers a_1, a_2,...,a_k, b, *and* n *such that*

$$a_1^n + a_2^n + ... + a_k^n = b^n,$$

then $k \geq n$.



A problem open for almost 200 years

Solving maths problems with a computer

Lander and Parkin (1966)

Solving maths problems with a computer

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$$27^5 + 84^5 + 110^5 + 133^5 = 144^5$$

Solving maths problems with a computer

COUNTEREXAMPLE TO EULER'S CONJECTURE ON SUMS OF LIKE POWERS

BY L. J. LANDER AND T. R. PARKIN

Communicated by J. D. Swift, June 27, 1966

A direct search on the CDC 6600 yielded

$$27^5 + 84^5 + 110^5 + 133^5 = 144^5$$

as the smallest instance in which four fifth powers sum to a fifth power. This is a counterexample to a conjecture by Euler [1] that at least n nth powers are required to sum to an nth power, $n > 2$.

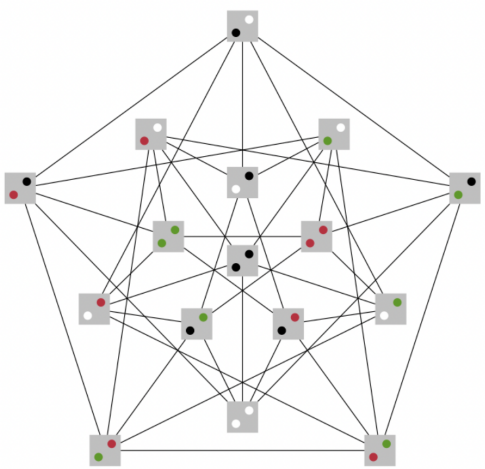
REFERENCE

1. L. E. Dickson, *History of the theory of numbers*, Vol. 2, Chelsea, New York, 1952, p. 648.

Solving maths problems with a computer

Proof of Keller's Conjecture (Brakensiek, Heule, Mackey, Narváez, 2019)

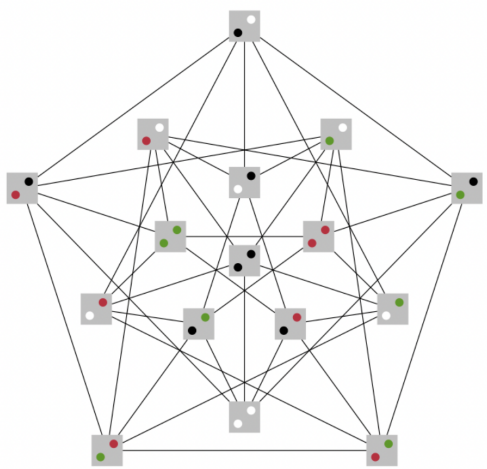
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- Many = far too many for a human



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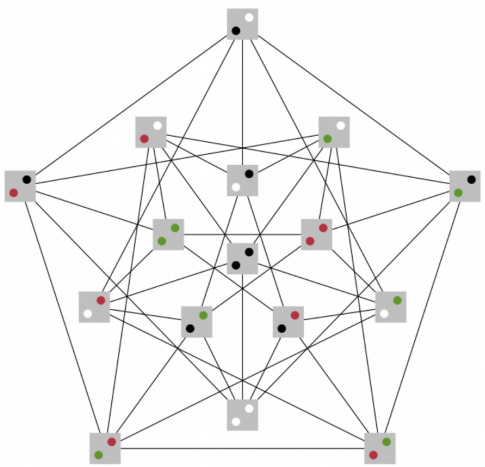
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Solving maths problems with a computer

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- Many = far too many for a human
- Size of the proof: 200Gb $\sim$ 10 Wikipedias



Conclusion: Computers have been used to prove theorems for a long time.

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Can AI be useful to solve more complicated problems?

Problems where the difficulty is not just a high number of case-checking?

AI in Mathematics Today

Three examples

- Stability of dynamical systems
- Control theory
- Combinatorics

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Stability of Dynamical Systems

A system of differential equations

$$\dot{x}(t) = f(x(t)),$$

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Question (System Stability)

Is it true that for every $\varepsilon > 0$, there exists $\delta > 0$ such that if the initial condition satisfies $\|x(0)\| \leq \delta$ then the solution $x(t)$ exists for all $t \in [0, +\infty)$ and

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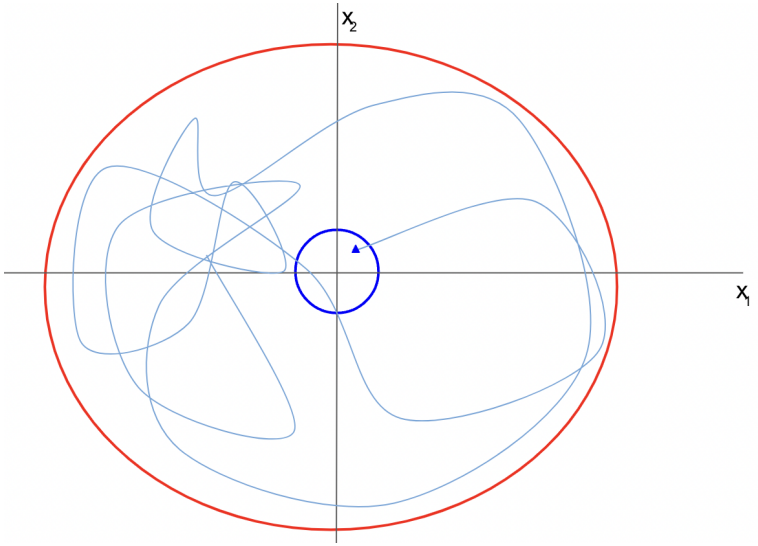
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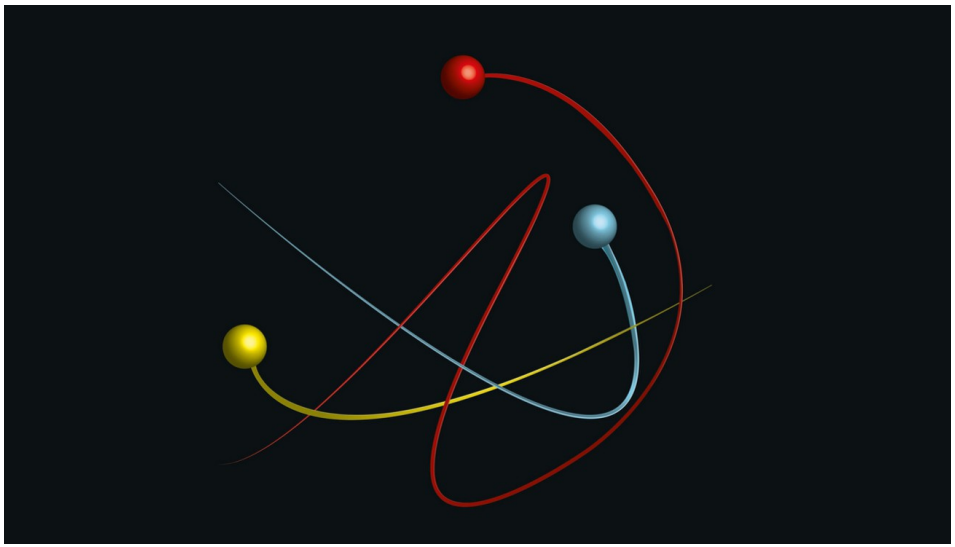
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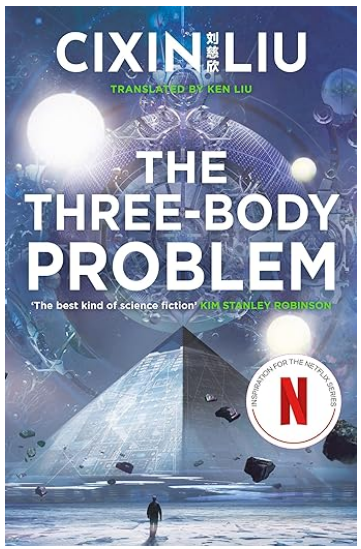
Stability of Dynamical Systems

A problem that has interested mathematicians for over a hundred years.





Stability of Dynamical Systems





Stability of Dynamical Systems

A significant advancement: Lyapunov functions

Theorem

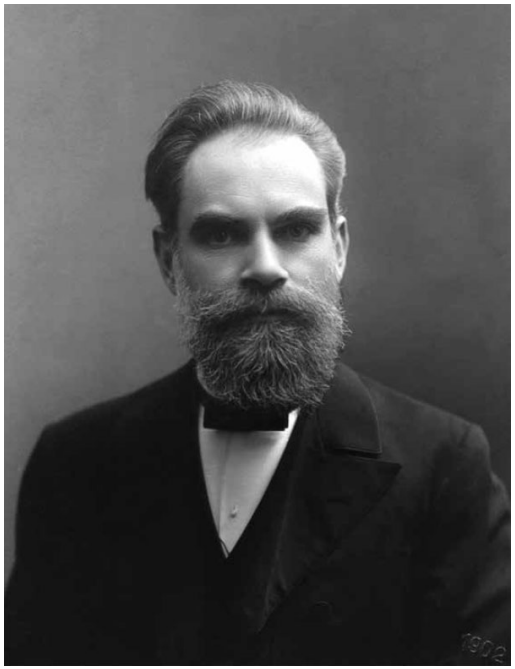
If there exists a function $V \in C^1(\mathbb{R}^n; \mathbb{R})$ *such that for all* $x \in \mathbb{R}^n$

$$V(x) > V(0), \quad \textit{and} \quad \nabla V(x) \cdot f(x) \leq 0,$$

and

$$\lim_{\|x\| \to +\infty} V(x) = +\infty,$$

then the system is stable.



A. Lyapunov
(1857-1918)

Stability of Dynamical Systems

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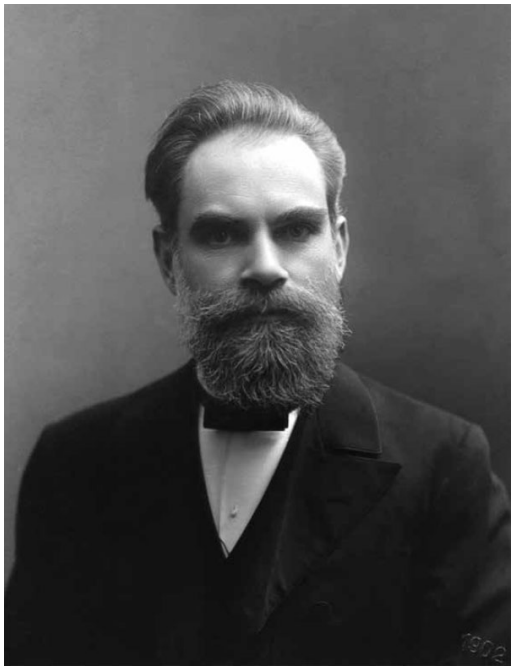
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Stability of Dynamical Systems

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Stability of Dynamical Systems

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The system is stable, a Lyapunov function is

$$V(x) = x_1^6 + 2(x_2^6 + x_3^4)$$

Stability of Dynamical Systems

A globally asymptotically stable polynomial vector field with no polynomial Lyapunov function

Publisher: IEEE

Cite This

PDF

Amir Ali Ahmadi ; Miroslav Krstic ; Pablo A. Parrilo **All Authors**

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Stability of Dynamical Systems

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there is still no systematic way to construct a Lyapunov function.

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$\rightarrow$ We resort to intuition

Stability of Dynamical Systems

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Task: guessing Lyapunov functions

$$\dot{x}(t) = \begin{pmatrix} -6x_1^4(t)x_2^5(t) - 3x_1^7(t)x_3^2(t) \\ 3x_1^9(t) - 6x_1^2(t)x_2^5(t)x_3^2(t) \\ -4x_1^2(t)x_3^5(t) \end{pmatrix} \quad \rightarrow \quad \text{Yes, } V(x) = x_1^6 + 2(x_2^6 + x_3^4)$$

Stability of Dynamical Systems

Train an AI to have an intuition of Lyapunov functions
(Alfarano, Charton, A.H., 2024).

Neural network architecture: Transformer ($\sim$1000 smaller than GPT-3)

Procedure:

1. Generate a set of systems and associated Lyapunov functions.
2. Encode the examples
3. Train the language model (supervised learning)

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- Find a mathematical way to get:

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  Then sample at random.

- In spirit: finding a Lyapunov function is a hard problem, checking that a function is a Lyapunov function is easier. NP-ish flavor in some sense.

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Limitations: even with a perfect generator, it biases the distribution (and it matters).

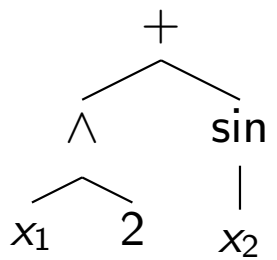
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$(x_1^2+\sin(x_2)) \quad \rightarrow$  $\rightarrow \quad$ "+","∧","x_1","2","sin","x_2"

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Stability of Dynamical Systems

Results

It works! The AI learns a mathematical intuition of Lyapunov functions.

[1] Existing method for some polynomial systems.

Stability of Dynamical Systems

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polynomial (train distrib.)	2-5	15%	**99%**
polynomial (fwd distrib.)	2-3	47%	**84-93%**
Non-polynomial	2-3	$\sim$ 0%	**87%**

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Master students accuracy: $\sim$ 10%

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My accuracy (!): $\sim$ 25%

[2] Existing method for some polynomial systems

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Meta says its AI could help mathematicians

Tada Images/Shutterstock

An AI system developed by Meta can find solutions to maths problems that have eluded mathematicians for over a century, researchers at the firm claim.

The problems involve mathematical tools called Lyapunov functions, named after mathematician Aleksandr Lyapunov, which analyse whether a system will remain stable over time, meaning its behaviour can be predicted. One famous example of such a system is the motion of three celestial bodies as a result of their mutual gravitational interactions – describing the behaviour of this "three-body problem" is extremely challenging.

Read more Incredible maths proof...



AIがリアプノフ関数の発見に革新！ 安定性理論に新たなブレイクスルー（2024-10）【論文解説シリーズ】

Summary of the approach

Paradigm: Training a Transformer to have a mathematical intuition on a problem

Key points:

- Generating data in a backward fashion (solution $\rightarrow$ problem)
- Test out-of-distribution on "real" instances of the problem.

Used in many frameworks: explicit solutions to ODE; local controllability; eigenvalues of random matrices; GCD; equilibrium of bio-networks, etc.

Try it yourself:

`https://github.com/ahayat16/Lyapunov/`

and customize on your favorite math problem

AI in Mathematics today

Three examples

- Stability of dynamical systems
- Control theory
- Combinatorics

Control of a Differential System

Evolution of the mosquito population

$$\begin{cases} \dot{E} = \beta_E F\left(1 - \dfrac{E}{K}\right) - (\nu_E + \delta_E)E, \\ \dot{M} = (1-\nu)\nu_E E - \delta_M M, \\ \dot{F} = \nu\nu_E E \dfrac{M}{M + M_s} - \delta_F F, \\ \dot{M}_s = u - \delta_s M_s, \end{cases}$$

$E(t)$ represents mosquito eggs, $F(t)$ fertilized females, $M(t)$ males, $M_s(t)$ sterile males. u is the flow of sterile mosquitoes that we release. This is what is called *control*.

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$$u = f(M + M_s, F + F_s)$$

with $F_s = FM/M_s$.

Control of a Differential System

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Question

Is it possible to find f such that the system is stable and

$$\lim_{t \to +\infty} \|E(t), M(t), F(t)\| = 0 \text{ and } \lim_{t \to +\infty} \|M_s(t)\| = \varepsilon,$$

with ε as small as desired?

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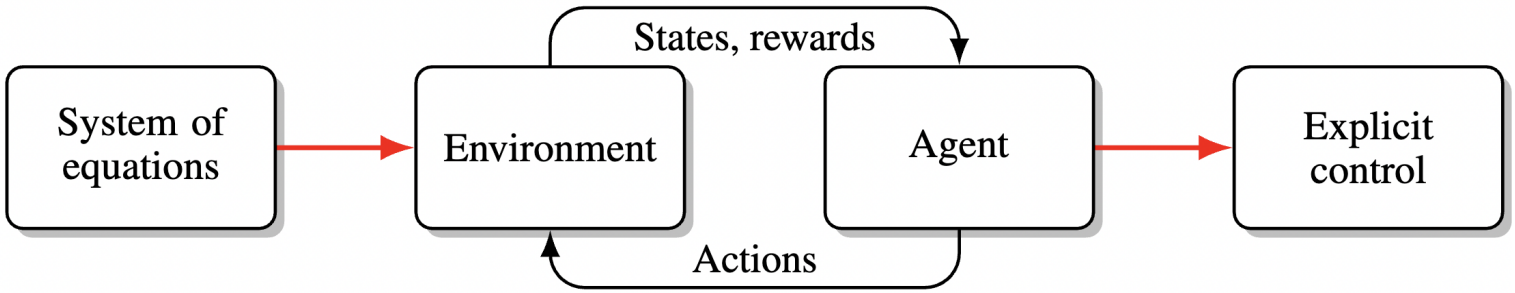
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An open question

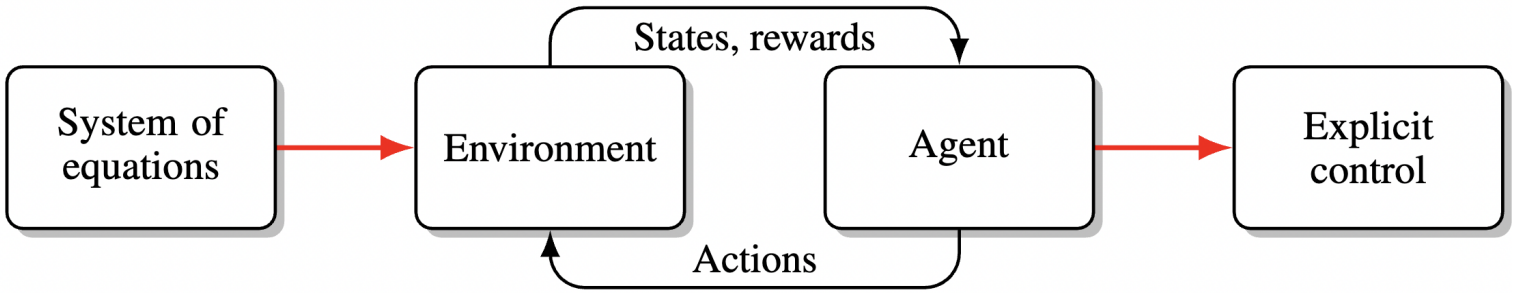
Control of a Differential System

Principle of the approach (Agbo Bidi, Coron, A.H., Lichtlé, 2023)



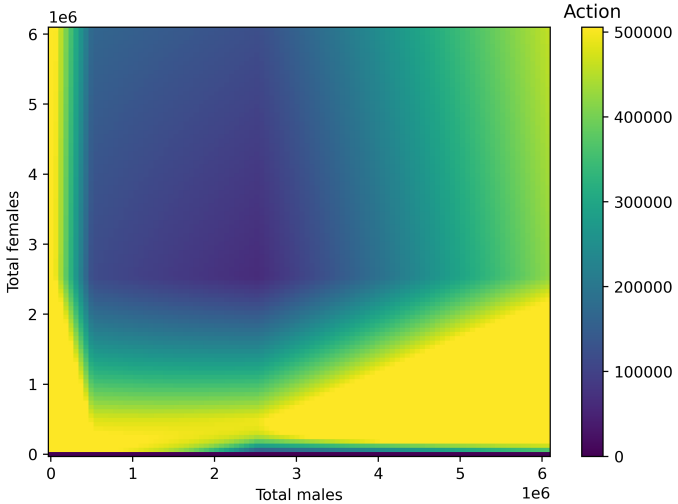
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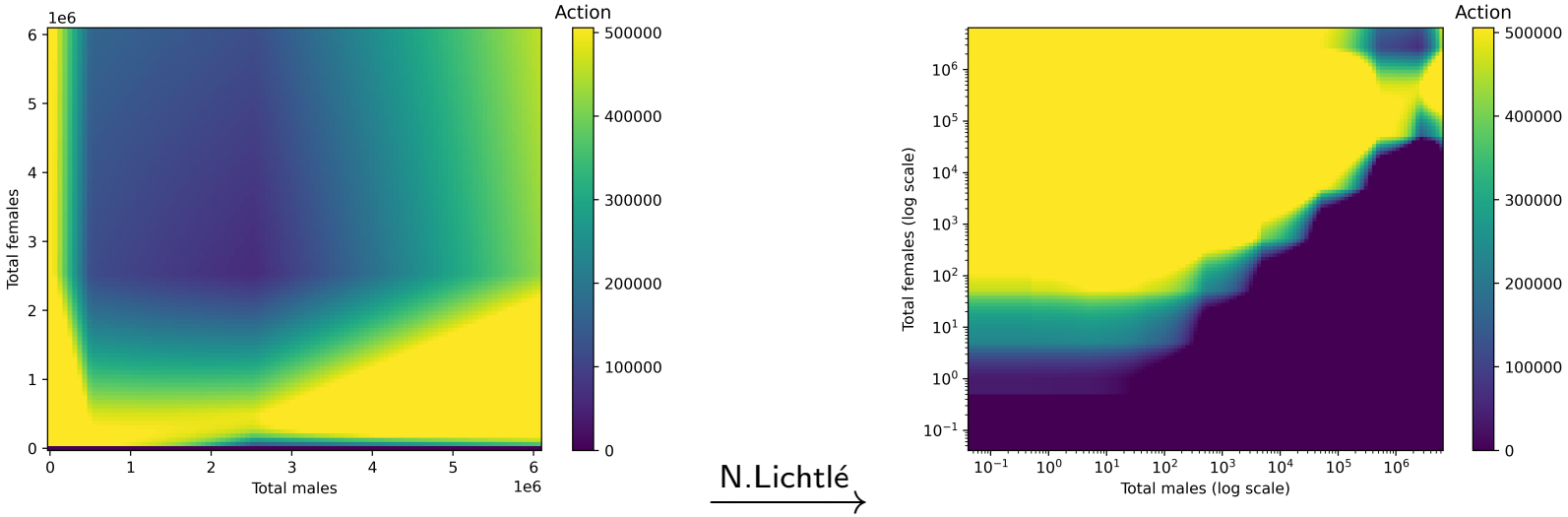
1. **Transform the equations** using a well-chosen numerical scheme.
2. **Train a Reinforcement Learning (RL) model**. The AI trains by trial and error and tries to maximize a well-chosen objective.
3. **Deduce the mathematical control**, from the numerical control.
4. **Verify** that it is a solution to the problem.

Control of a Differential System



$$u = f(M + M_s, F + F_s)$$

Control of a Differential System



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Control of a Differential System

$$u_{\text{reg}}(M+M_s, F+F_s) = \begin{cases} u_{\text{reg}}^{\text{left}}(M+M_s, F+F_s) & \text{if } M+M_s < M^*, \\ u_{\text{reg}}^{\text{right}}(M+M_s, F+F_s) & \text{otherwise,} \end{cases}$$

$$u_{\text{reg}}^{\text{left}} = \begin{cases} \varepsilon & \text{if } l_1(F+F_s) > \alpha_2, \\ u_{\max}\,(\alpha_2 - l_1) & \text{if } l_1 \in (\alpha_1, \alpha_2], \\ u_{\max} & \text{otherwise, and} \end{cases}$$

$$u_{\text{reg}}^{\text{right}} = \begin{cases} \varepsilon & \text{if } l_2 > \alpha_2, \\ u_{\max}\,(\alpha_2 - l_2) & \text{if } l_2 \in (\alpha_1, \alpha_2], \\ u_{\max} & \text{otherwise.} \end{cases}$$

where $l_1(x) = \frac{\log M^*}{\log(F+F_s)}$ and $l_2(x,y) = \frac{\log(M+M_s)}{\log(F+F_s)}$,

Control of a Differential System

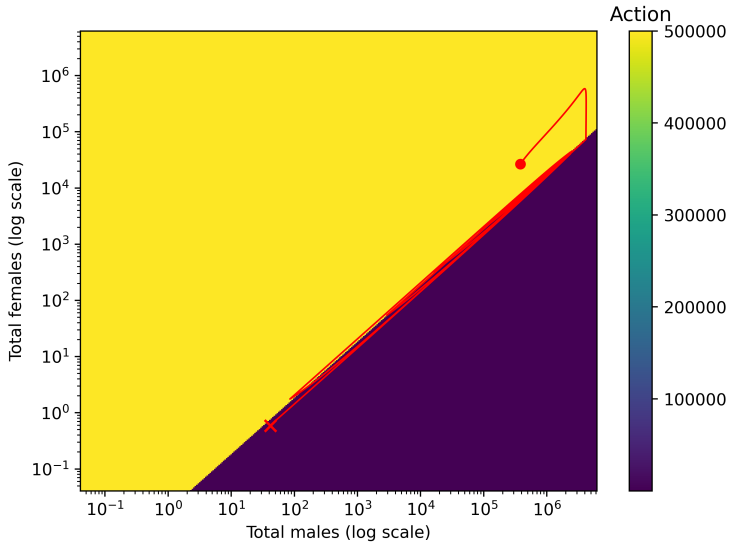
Final control

$$u(t) = \begin{cases} \varepsilon & \text{if } \frac{\log(M+M_s)}{\log(F+F_s)} > \alpha_2, \\ u_{\max} & \text{otherwise,} \end{cases}$$

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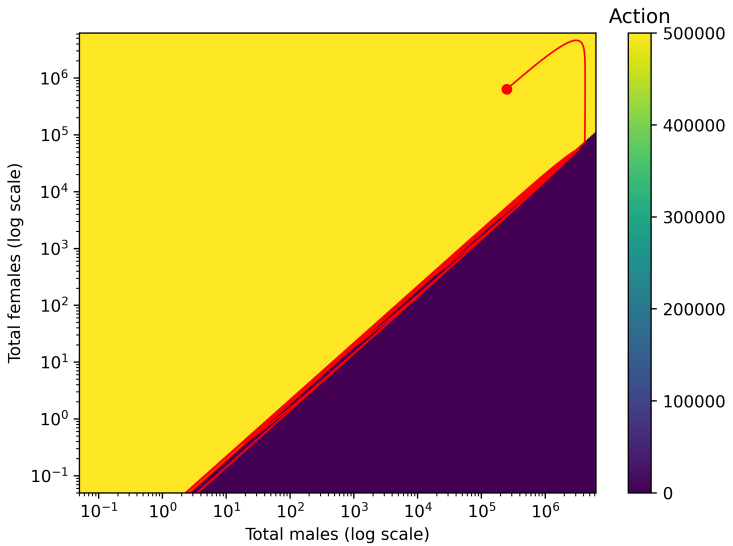


$$\varepsilon > 0$$

Control of a Differential System

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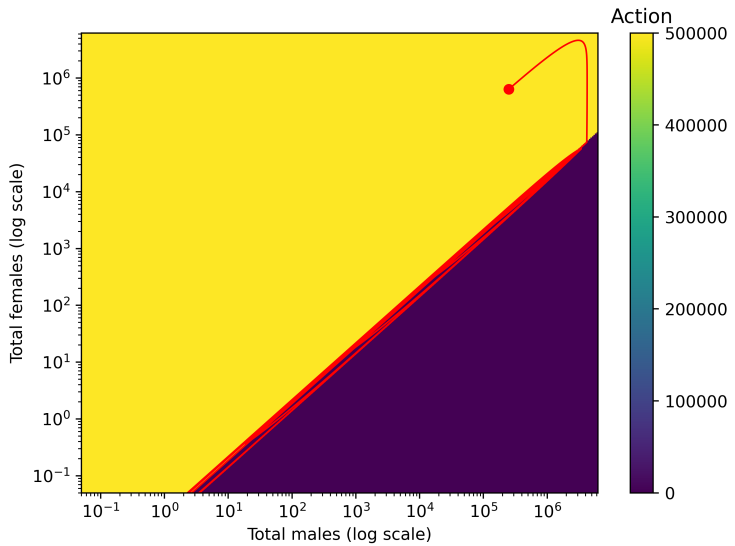


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$\varepsilon = 0$

We can see a mathematical bifurcation with our "AI-augmented intuition".

Control of a Differential System

Robustness of the solution

Can the solution still work if there are uncertainties on the parameters ?

$$\begin{cases} \dot{E} = \beta_E F \left(1 - \dfrac{E}{K}\right) - (\nu_E + \delta_E) E, \\ \dot{M} = (1-\nu)\nu_E E - \delta_M M, \\ \dot{F} = \nu\nu_E E \dfrac{M}{M + M_s} - \delta_F F, \\ \dot{M}_s = u - \delta_s M_s, \end{cases}$$

$$\begin{aligned} \hat{\beta}_E &\sim \mathcal{U}(0.65, 9.03) \\ \hat{\nu}_E &\sim \mathcal{U}(0.01, 0.35) \\ \hat{\delta}_E &\sim \mathcal{U}(0.03, 1.0) \\ \hat{\delta}_F &\sim \mathcal{U}(0.03, 0.1) \\ \hat{\delta}_M &\sim \mathcal{U}(0.09, 0.2) \\ \hat{\nu} &\sim \mathcal{U}(0.39, 1.0) \end{aligned}$$

Yes

AI as a tool for math discovery

Three examples

- Stability of dynamical systems
- Control theory
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PatternBoost: finding interesting constructions

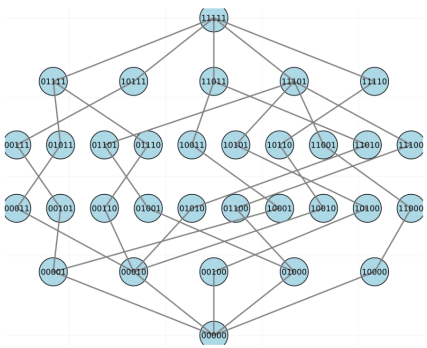
Paradigm: couple a classical algorithm and a transformer. [Charton, Ellenberg, Wagner, Williamson, 2024]

Step 1: a classical algorithm generate "good" constructions from random seeds

Step 2: train a transformer on the best such constructions

Step 3: use the transformer as seeds.

Repeat.



PatternBoost: finding interesting constructions

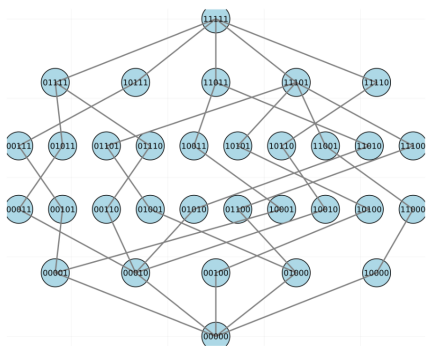
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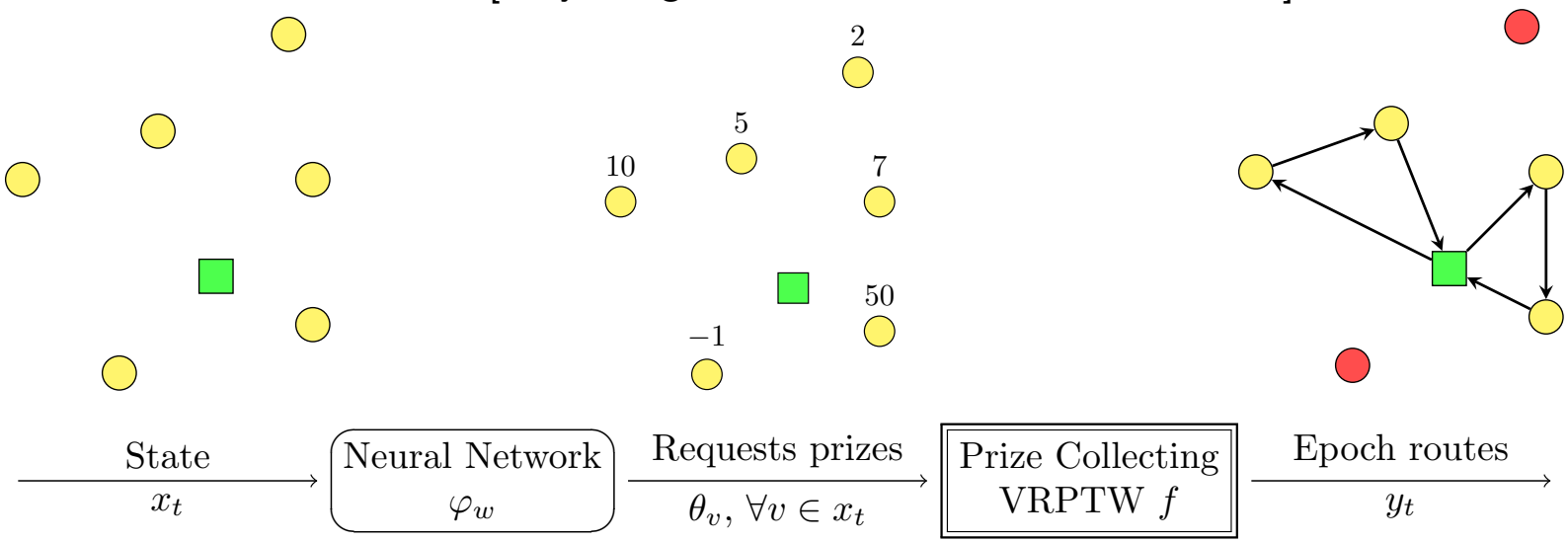
Repeat.



In particular, closed a 30-years conjecture

Combinatorial Optimization Augmented Machine Learning

Paradigm: couple a combinatorial optimization algorithm and a neural network and train the ensemble [Baty, Jungel, Klein, Parmentier, Schiffer, 2024].



Perform better than existing mathematical algorithm and machine learning algorithm

(winner of EURO NeurIPS challenge 2022)

Many other examples

- in topology feedforward and MPNNs to guess links between different mathematical quantities, in particular hyperbolic and algebraic invariant of knots (a conjecture that was later proved) [Davies et al., 2021]
- in group theory path-finding for large graphs with ResMLP and RL [Chervov et al. 2025]
- in partial differential equations PINNs to find an exact self-similar solution to 3D Euler [Wang, Lai, Gómez-Serrano, Buckmaster, 2023]
- ... and in many others fields

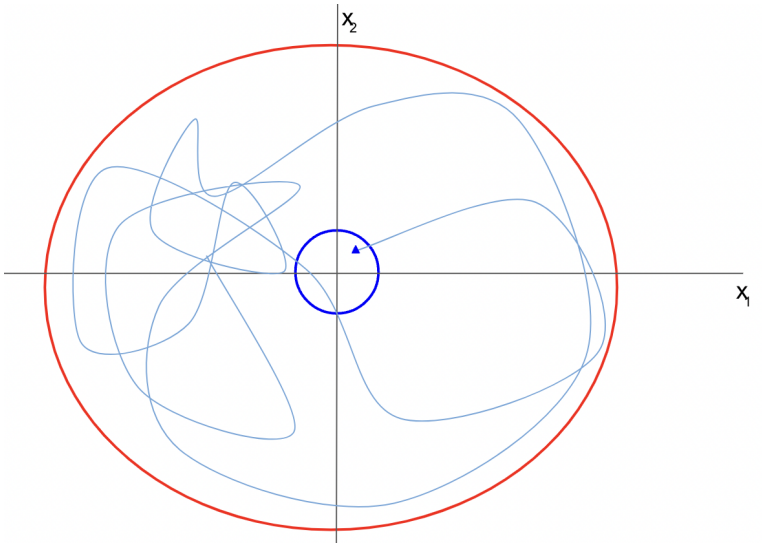
AI as a tool for math discovery

- AI are already useful in the practice of mathematics and can help solve difficult problems.
- AI are trained to have better intuition than humans on a specific problem.
- This augmented intuition allows us to bypass the difficulty of the problem.

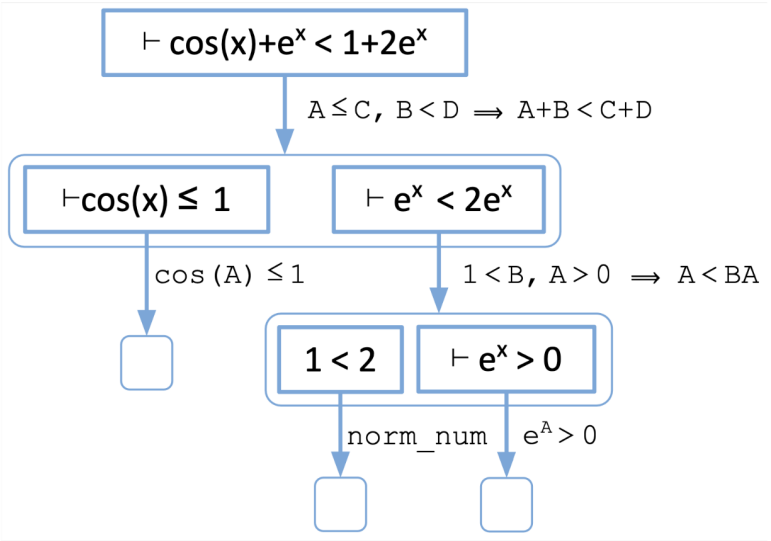
Future of Mathematical AI

Can an AI prove a mathematical result on its own?

Outline of the talk



1. AI as a tool for math discovery



2. When will AI prove theorems?

AI for Mathematical Proof

Can a trained AI find a proof for a mathematical statement?

- A much harder problem
- Shocking question: calls into question our very vision of mathematics
- A science-fiction future that is probably closer than we imagine

AI for Mathematical Proof

First approach: training a Transformer (GPT-f, Polu, Sutskever, 2020)

Question

Let $a > 0$ and $b > 0$, such that $ab = b - a$, show that

$$\frac{a}{b} + \frac{b}{a} - ab = 2$$



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Ilya Sutskever: The OpenAI Genius Who Told Sam Altman He Was Fired

Company's chief scientist led a board coup against one of the most prominent figures in Silicon Valley



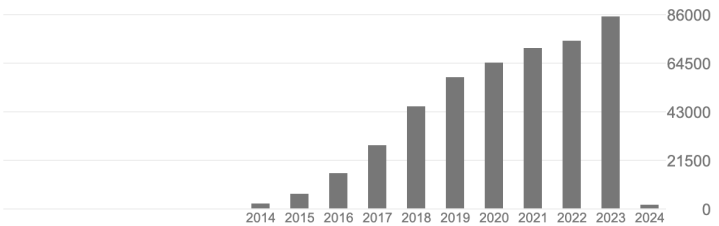
Ilya Sutskever SUIVRE

Co-Founder and Chief Scientist of OpenAI
Adresse e-mail validée de openai.com - Page d'accueil

Machine Learning Neural Networks Artificial Intelligence Deep Learning

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Citations	462122	356473
indice h	84	79
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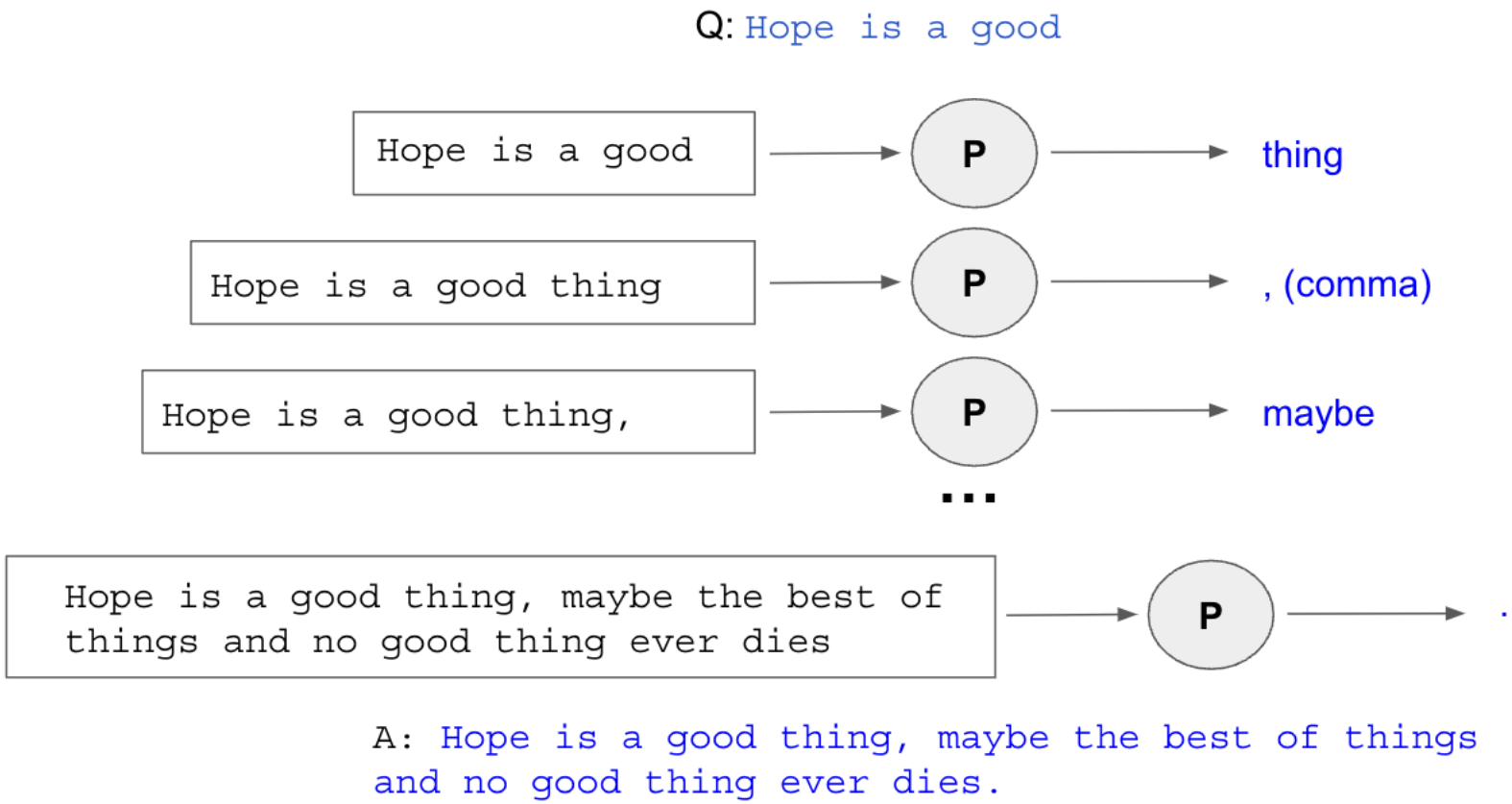
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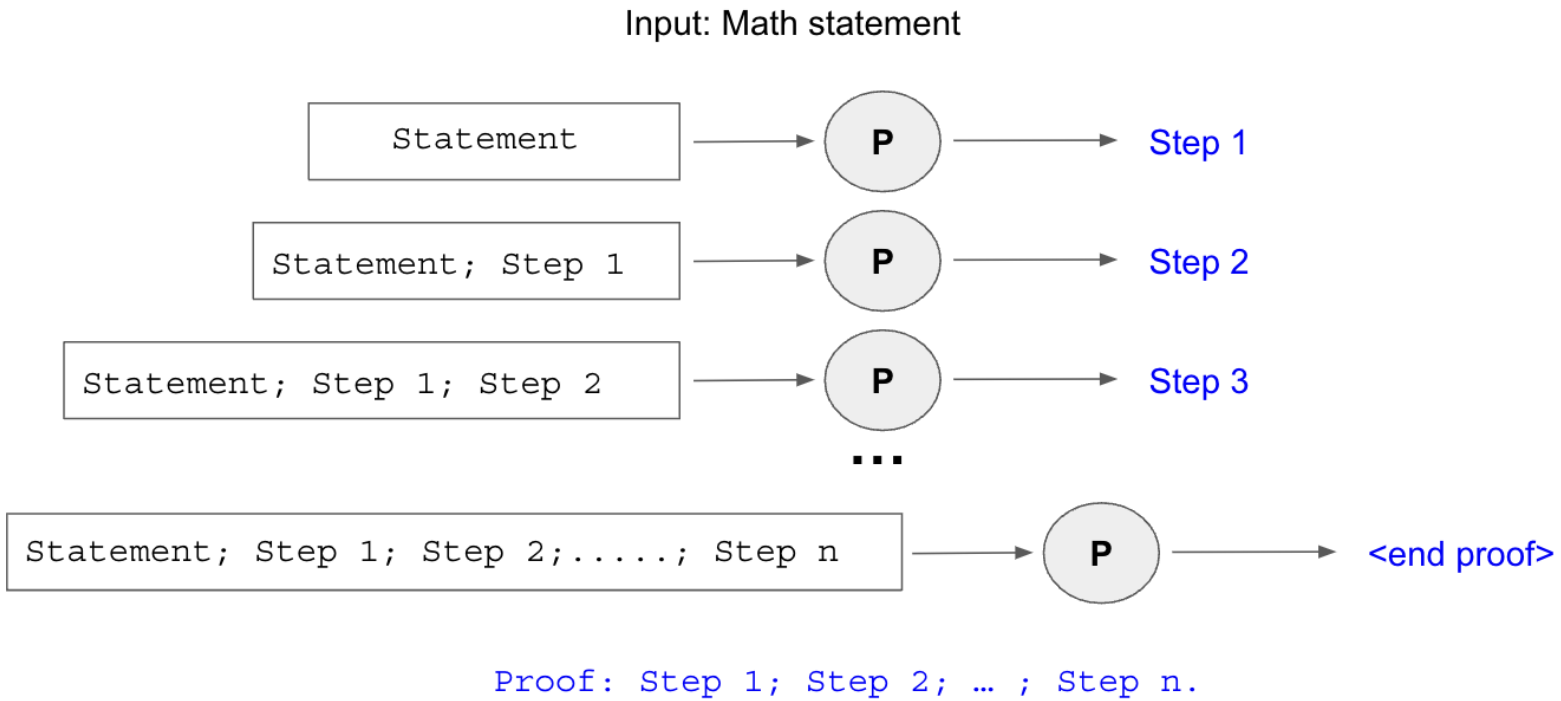


AI for Mathematical Proof

What does an autoregressive transformer do (in a nutshell) ?

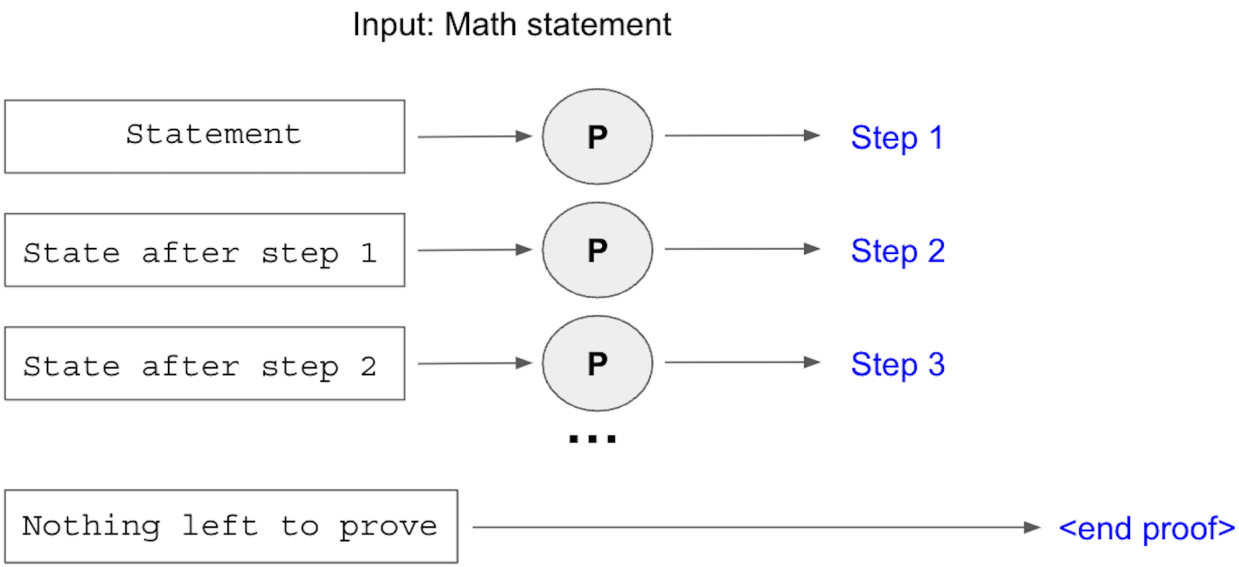


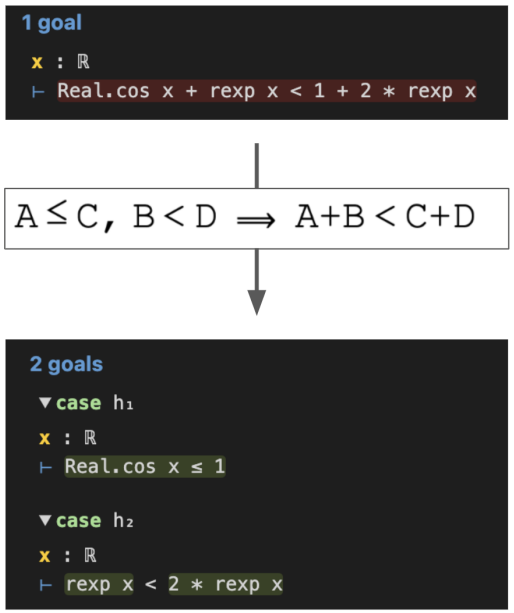
AI for Mathematical Proof



Proof: Step 1; Step 2; … ; Step n.

AI for Mathematical Proof





AI for Mathematical Proof

```
theorem Exercice_1
(a b : ℝ)
(h₀: a > 0)
(h₁: b > 0)
(h₂: a*b = b-a) :
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begin
sorry,
end
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Procedure: train it with examples: (exercises, proofs)

- The hope is that by showing it enough examples, the AI will be capable of learning to reason, just by learning to predict the next step each time.

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Enough = sufficiently diverse and sufficiently numerous

→ Limitation: lack of data

AI for Mathematical Proof

We have very few data available (especially formal).

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Lean: ∼300,000 theorems. A large dataset for humans, a small dataset for AI. → Limit of the approach

AI for Mathematical Proof

How to tackle this limit ?

Many subsequent improvements and variations

- Training a retriever on the library to suggest relevant theorems: **LeanDojo** (Yang et al., 2023)
- Fine-tuning better base LLM: **LeanLlama** (Glöckle et al., 2023)

Tackle the lack of data

- Using additional data gathered on the internet: **Llemma** (Azerbayev et al., 2023), **InternLM-2** (Wu et al. 2024)
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AI for Mathematical Proof

Autoformalization: a natural idea to get more formal data → train a model to translate from "natural language proof" to formal and verifiable data.

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formal language

Autoformalization: both a means and an end:

- a means: more data to train AI models
- an end: checking the correctness of new mathematical theories

AI for Mathematical Proof

Autoformalization: both a means and an end

Example of contradictory papers:

- Schumacher, G., & Tsuji, H. (2004). Quasi-projectivity of moduli spaces of polarized varieties. Annals of mathematics, 597-639.
- Kollár, J. (2006). Non-quasi-projective moduli spaces. Annals of mathematics, 1077-1096.

A quickly improving field:

- (2024) Very good statement autoformalization for olympiad style exercise (e.g. Herald, Numina, AlphaProof)
- (2025) 4% of statements in arxiv papers autoformalized automatically in most field of mathematics
- (2025) a first arxiv paper completely autoformalized by Morph Labs (proofs included) with little human intervention

Still challenges for statement and proof autoformalization in most fields of mathematics

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Talk of Wenda Li this afternoon

AI for Mathematical Proof

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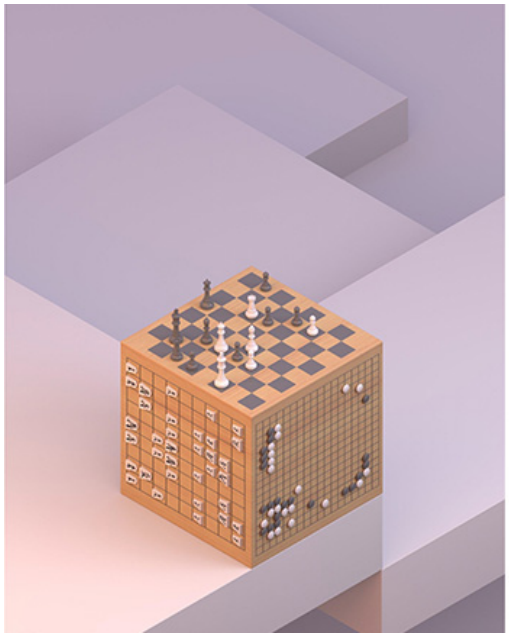
How to go further?

AI for Mathematical Proof

Second approach: **treat mathematics as a game** (Lample, Lachaux, Lavril, Martinet, Hayat, Ebner, Rodriguez, Lacroix, 2022);

AI for Mathematical Proof

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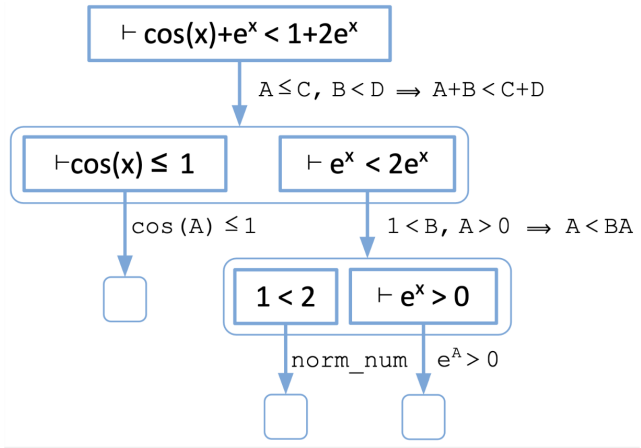
Deepmind (2017)

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Second approach: **treat mathematics as a game** (Lample, Lachaux, Lavril, Martinet, Hayat, Ebner, Rodriguez, Lacroix, 2022);



Deepmind (2017)

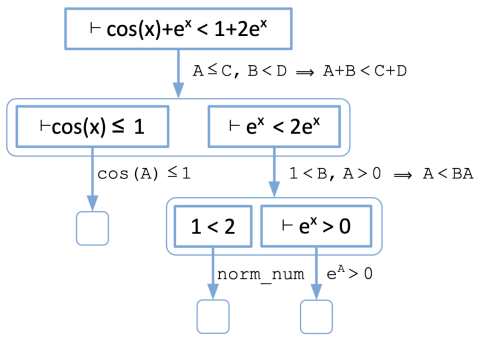


You won !

AI for Mathematical Proof

Main difficulties:

- two-player game vs. solo against a goal.
- In chess, when you play a move you always have a single game. In mathematics: one statement $\longrightarrow$ multiple statements
- Difficult in mathematics to know automatically in the middle of a proof what the probability of succeeding is.
- The number of possibilities is much, much larger in mathematics

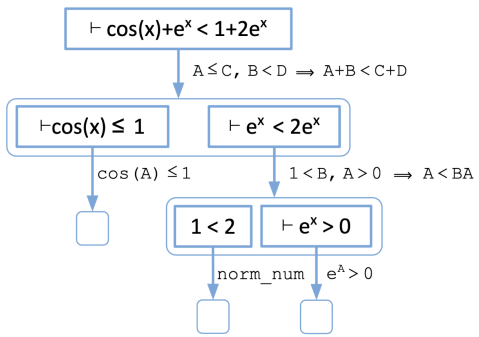


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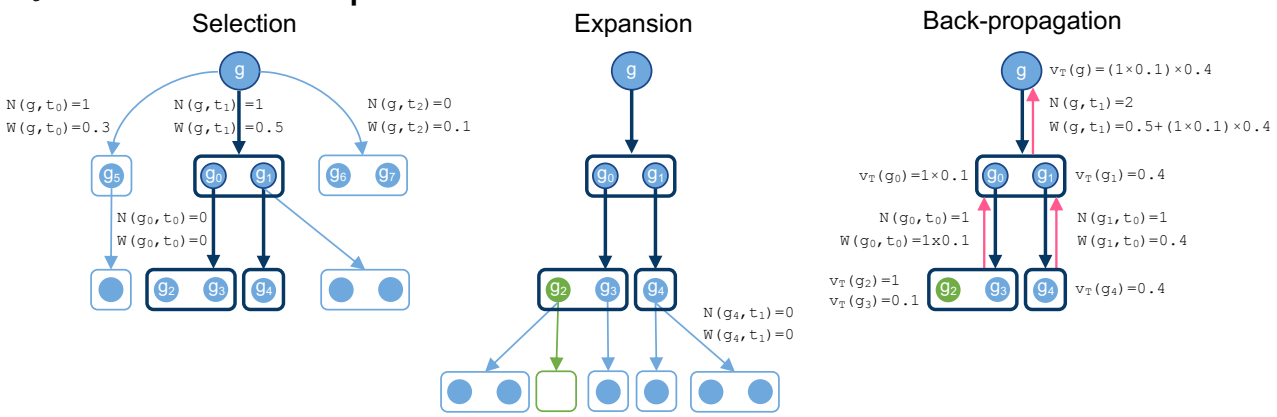
Much more difficult than chess

AI for Mathematical Proof

In practice

- Two transformers: P_θ which predicts a tactic, c_θ which predicts the difficulty of proving a statement (goal, hypothesis, etc.).

- An intelligent proof search that sees the proof as a tree and combines P_θ, c_θ and a tree expansion.



- Continuously training of P_θ and c_θ on successful proofs

AI for Mathematical Proof

Results

Exercises at the undergraduate level...

...60% of middle school / high school exercises up to Olympiad level...

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Show that for all $n \in \mathbb{N}$, 7 does not divide $2^n + 1$.

AI for Mathematical Proof

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...60% of middle school / high school exercises up to Olympiad level... More recent approach reached $90 - 99\%$ of success (Deepseek, Kimina, Seed, 2025)

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More recent approach reached a gold medal level at the International Mathematical Olympiads (Seed, 2025)

Theorem proving

ABEL: Hypertree proof search 2.0 (Glöckle, Limperg, A.H., Synnaeve, 2024)

ABEL:Sample Efficient Online Reinforcement Learning for Neural Theorem Proving

A better AlphaZero style proof-search in the era of pre-trained models:

- on-par with state of the art on high-school exercise (in Oct. 2024)
- state-of-the-art on PutnamBench for 3,5 months
- 20x less compute than HTPS
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Should be open-sourced soon!

AI and maths

A very (very) fast-moving field

2019	2022	2024
$(r_1 - r_2)r_3 = r_1 r_3 - r_2 r_3$	$\forall\, n \in \mathbb{N},\ \neg 7 \mid 2^n + 1$	Silver medal Inter. Math. Olymp.
HOList, LPLG	GPT-f, Thor/DSP, HTPS	Llemma, LeanDojo, InternML, DeepSeek, ABEL, AlphaProof, etc. 2025 wouldn't fit on the slide

New approaches post 2024:

- end-to-end RL and whole proof generation
- multiturn proving with execution feedback
- Sketch then prove (see Talk of Wenda Li)

Conclusion

- AI methods are already useful in the practice of mathematics
- AI and LLM for proving theorems is only beginning, and there are many ideas... and much to do.
- LLMs will not be the final AI tool which will be used to mathematics.
- The practice of mathematics will probably change... and that's okay.
- AI will not replace mathematicians but will instead enhance them.

Conclusion

Thank you for your attention