
***Workshop on Partial Differential Equations and
Calculus of Variations***
Second Annual Meeting of the GAMM Activity Group
October 5-7, 2026



Scientific Scope

Framed within the Activity Group Analysis of Partial Differential Equations and Calculus of Variations of GAMM (International Association of Applied Mathematics and Mechanics), this workshop addresses researchers interested in analysis of partial differential equations and calculus of variations in the context of mechanics problems. New developments and model classes in applications usually lead to new classes of partial differential equations or variational problems and new mathematical questions.

On the one hand, a rigorous understanding of the mathematical properties is important for the qualitative evaluation of the models. On the other hand, it is the basis for the development of efficient and reliable numerical methods and the treatment of concrete problems in applications. This workshop will close the gap between mathematicians and more applied researchers. We aim to identify interesting research questions with high practical relevance.

Invited Speakers

- Alexandra Holzinger (Utrecht University)
- Virginie Ehrlacher (Institut Polytechnique de Paris & INRIA)
- Ben Schweizer (TU Dortmund)
- Markus Gahn (University of Augsburg)
- André Schlichting (Ulm University)
- Leonard Kreutz (TU Munich)
- Sonia Fliss (ENSTA Paris)
- Annika Bach (Eindhoven University of Technology)

Program

Time	Monday	Tuesday	Wednesday
08:30 - 09:10		Ben Schweizer	Sonia Fliss
09:10 - 09:30		Gianna Götzmann	David Wiedemann
09:30 - 09:50		Guy Foghem	Wolfgang Reichel
09:50 - 10:10		Hidde Schönberger	Nicola Zamponi
10:10 - 10:40		Coffee Break	Coffee Break
10:40 - 11:20		Markus Gahn	Annika Bach
11:20 - 11:40		Dominik Engl	Francesco De Anna
11:40 - 12:00		Lucas Fix	Alex Suri
12:00 - 12:20		Joshua Kortum	Leonardo Del Grande
12:20-13:20	Registration and Welcome	Lunch	
13:20 - 14:00	Alexandra Holzinger		
14:00 - 14:40	Virginie Ehrlicher	André Schlichting	
14:40 - 15:00	Illia Karabash	Leonard Kreutz	
15:00 - 15:20	Kateryna Buryachenko		
15:20 - 15:50	Coffee Break	Coffee Break	
15:50 - 16:10	Christian Wagner	Noah Piemontese	
16:10 - 16:30	Christopher Körber	Timo Hofmann	
16:30 - 16:50	Lennart Machill	Timo Ziereis	
16:50 - 17:10		Felix Seifert	
17:10 - 17:30	CAAPS Symposium Building T 17:15	General Meeting 17:30	
19:00		Conference Dinner	

Coffee break

All scheduled talks will be held in room 2107, building D. Adjacent to it, the seminar room 2108 will serve as the catering area for coffee breaks as well as an informal space for further discussions divided into smaller groups.

Lunch

Reservations have been made on **October 6 at 12:00 p.m.** at the following two restaurants. Each has a table reserved for up to 20 people:

- Restaurant Unikum, Salomon-Idler-Straße 24 F
- Restaurant IL Porcino, Salomon-Idler-Straße 24B

Lunch is however not provided, and participants are responsible for their own expenses.

Dinner

Join us for the conference dinner on **October 6 at 7:00 p.m.** at **Riegele Wirtshaus**. Dinner and water are provided free of charge; additional drinks are at one's own expense. The address is Frölichstr. 26, 86150 in Augsburg.

Abstracts

Alexandra Holzinger (Utrecht University)

Moderately interacting particles and their macroscopic PDE limits

In this talk, I will start by giving a short introduction to mean-field limits and their limitations when deriving PDEs from systems of interacting particles.

In the classical (weak interaction) mean-field regime, each particle—modeled by a stochastic differential equation (SDE)—interacts with every other particle with a strength scaling like $1/N$, where N is the number of particles. Under suitable assumptions, this leads to propagation of chaos, and the empirical distribution of the particles converges to a deterministic density solving a McKean–Vlasov equation (a non-linear PDE).

However, this weak interaction regime does not recover many important PDEs arising in physics and biology—such as the porous medium equation.

I will explain how increasing the interaction strength slightly—beyond the classical $1/N$ scaling, but still below order one—modifies the limiting dynamics. This scaling regime is called moderate interaction regime.

Moreover, I will give insights on my recent result on fluctuations around the mean-field limit in the moderate regime. This can be seen as a Central Limit Theorem for interacting particles and gives a stochastic correction to the PDE limit.

This talk is based on articles together with Ansgar Jüngel (TU Wien), Li Chen (University of Mannheim), José A. Carrillo (University of Oxford) and Shuchen Guo (University of Oxford).

Virginie Ehrlacher (Institut Polytechnique de Paris & INRIA)

Structure-Preserving Hybrid Finite Volume Methods for Anisotropic Cross-Diffusion on General Meshes (joint work with Annamaria Massimini and Julien Moatti)

Anisotropic cross-diffusion systems arise as thermodynamic limits of multi-species stochastic exclusion processes and provide mesoscopic models for diffusion in solid mixtures. Their numerical approximation is particularly challenging, since nonlinear coupling, anisotropy, and size-exclusion constraints must be handled without destroying the structural properties of the continuous model.

In this talk, we introduce a nonlinear hybrid finite volume scheme defined on general polygonal and polyhedral meshes. The method is formulated in terms of discrete entropy variables, allowing the entropy structure of the underlying system to be preserved at the discrete level. We establish the existence of nonnegative discrete solutions satisfying the size-exclusion constraint, as well as mass conservation and a discrete entropy estimate. We then prove the convergence of the scheme under mesh refinement.

A key advantage of the hybrid formulation is its applicability to general, possibly non-orthogonal meshes, thereby avoiding the geometric restrictions associated with standard two-point flux approximation methods. To the best of our knowledge, this is the first structure-preserving hybrid finite volume scheme analyzed for anisotropic cross-diffusion systems on general polygonal and polyhedral meshes.

Joint work with Annamaria Massimini and Julien Moatti.

Illia Karabash (University of Bonn)

Sobolev multipliers on non-smooth boundaries and randomization of boundary conditions

Motivated by Applied Physics and Photonics studies of random resonators, we plan to demonstrate an approach to random acoustic operators in bounded domains involving stochastic lossy boundary

conditions. In particular, m -dissipative impedance boundary conditions containing (eigenfunction-based) fractional Gaussian fields will be rigorously introduced. As the main deterministic tool, we construct and study spaces of pointwise multipliers on Lipschitz boundaries, as well as spaces of Sobolev (distribution-type) multipliers on boundaries of better regularity. These multipliers are used as generalized impedance coefficients in impedance boundary conditions accompanying the first-order acoustic system. We study the m -dissipativity of the associated acoustic operators and the discreteness of their spectra aiming the main efforts on the weakest possible assumptions on the regularity of impedance coefficients. In order to pass from deterministic results to the randomization, we will introduce fractional Gaussian fields (FGFs) on Lipschitz boundaries and consider their regularity. To this end, a rough Weyl-type asymptotics for the Laplace-Beltrami eigenvalues on an arbitrary compact Lipschitz boundary is needed, and we plan to discuss this asymptotics if time allows. The talk is based on the recent papers [doi.org/10.1016/j.jmaa.2025.129985] and [doi.org/10.48550/arXiv.2601.14600].

Kateryna Buryachenko (WIAS Berlin)

Double-phase anisotropic thermistor model with discontinuous variable exponents

We analyze a novel double-phase thermistor model for semiconductor devices. The model accounts for different temperature-dependent charge-transport mechanisms acting in parallel and coupled through a spatial modulation coefficient $a(x)$. The current-flow equation is governed by a weighted $(p(x), q(x))$ -double-phase operator, where the exponents p and q are allowed to be merely measurable. We prove the existence of solutions to the coupled system by means of Schauder's fixed-point theorem. For the current-flow subproblem, we work in Musielak–Orlicz spaces and in a related generalized Sobolev space adapted to the double-phase structure. Since the right-hand side of the heat equation is, a priori, only an L^1 -function, the thermal subsystem is treated in the framework of entropy solutions. Joint work with Annegret Glitzky and Matthias Liero.

Christian Wagner (ISTA)

Diffuse Interface Energies with Microscopic Heterogeneities: Homogenization and Rare Events

In this talk we discuss the Allen-Cahn functional with stationary and ergodic coefficients in the regime, where the length scale δ of the heterogeneities is much smaller (microscopic) than the interface width ϵ (mesoscopic). If the ratio $\epsilon^{-1}\delta$ vanishes fast enough as $\epsilon \rightarrow 0$, then the functionals converge to an effective surface energy where the energy density is determined by homogenization effects originating at the microscopic scales. When the ratio $\epsilon^{-1}\delta$ vanishes too slowly, the limit of the functional may be strictly smaller than this homogenized energy. The latter is called the rare events regime. In this talk, we analyse the random checkerboard in dimension one and use large deviations techniques to give a complete description of the rare events regime, showing that the limiting energy depends in a nontrivial way on the limit of $\epsilon^{-1}\delta |\log \epsilon|$. This behavior is in contrast to periodic homogenization, where the effective surface energy is obtained by first taking the limit $\delta \rightarrow 0$ followed by the limit $\epsilon \rightarrow 0$.

This is joint work with P. Morfe.

Christopher Körber (University of Würzburg)

Linearization of a variational model for fluid-structure interaction

We consider the interaction problem between a viscoelastic body and a surrounding incompressible viscous fluid. The system is contained in a fixed bounded domain of dimension bigger or equal to two,

and the solid is partly attached to the boundary. Beside the full system, we deal with a time-delayed approximation, where the highest order time derivatives are replaced by finite differences. Assuming smallness of external forces and initial data, we perform a linearization of both the full and the time-delayed problem. More precisely, in the limit of small data, we rigorously prove that the full and the time-delayed problem converge towards the corresponding linearized systems. We show this for both systems on the level of equations and for the time-delayed system also on the level of energies, by using the concept of EDP convergence. Further, we highlight the difficulties when applying EDP convergence to the full system, which is ongoing work.

This is joint work with Barbora Benešová (Charles University Prague).

Lennart Machill (University of Bonn)

Frame-indifferent Approximation in Nonlinear Thermoviscoelasticity

We consider a Kelvin-Voigt model for thermoviscoelastic second-grade materials, where the elastic and the viscous stress tensor both satisfy frame indifference. We prove the existence of weak solutions relying on an estimate due to Ciarlet and Mardare (2015), which generalizes the celebrated rigidity estimate of Friesecke, James, and Müller (2002). The estimate allows us to employ a staggered time-discretization scheme that preserves frame indifference, in which the deformation and the temperature are updated alternately.

The talk is based on joint works with Manuel Friedrich, Martin Kruzik, Rufat Badal, and Martin Horak.

Ben Schweizer (TU Dortmund)

Edge-modes at interfaces between periodic media

When two periodic media are separated by an interface, it is possible that the corresponding wave equation has solutions that are standing waves at the interface. In other words: It is possible that an eigenvalue appears inside a spectral gap when a 1-periodic medium is perturbed so that the resulting medium is only periodic on the two sides of the interface. The effect is related to the quantum-Hall-effect and has received enormous attention in physics since the 1980ies. A deeper understanding of the effect can be summarized with the following statement: A certain topological index is different on the two sides of the interface; this implies a non-trivial topological index for the interface and thus the existence of the interface eigenvalue. We present the effect and the necessary background. We prove mathematically the existence of the interface eigenvalue, our method is not topological, but perturbative. Homogenization theory is used to derive reduced spatial dynamics that characterize the (piecewise) periodic media. An analysis of the system of ordinary differential equations provides the proof for the spectral result.

Gianna Götzmann (University of Augsburg)

Perturbed Minimizing Movements of Time-Dependent Functionals on Metric Spaces

Minimizing movement schemes are a fundamental tool for approximating evolution equations that have a gradient-flow structure with respect to an energy or entropy functional $\mathcal{F}$. Motivated by homogenization problems involving, for instance, a geometry that evolves in time, we consider perturbations of the functional in the sense of a Γ -converging sequence $\mathcal{F}_{\varepsilon}$, as well as a time dependence. Accordingly, our minimizing movement scheme includes a time discretization $\mathcal{F}_{\varepsilon, \tau, n}$, $n=1, \dots, N$, of $\mathcal{F}_{\varepsilon}$ where N is the number of time steps and τ is the step size. We study the case where ε and τ tend to 0 simultaneously and establish sufficient conditions

ensuring the commutativity of the limits. This is joint work with Jan-Frederik Pietschmann and Antonio Tribuzio.

Guy Foghem (BTU Cottbus-Senftenberg)

Optimal stability of the Dirichlet boundary value problem for the regional fractional p -Laplacian

We establish the optimal stability of nonlinear Dirichlet boundary value problems on bounded Lipschitz domains involving the regional (fractional) p -Laplacian $(-\Delta)^s_{p,\Omega}$ for $0 < s \leq 1$ and $p > \frac{1}{s}$. More precisely, let $u_s \in W^{s,p}(\Omega)$ be a solution of $(-\Delta)^s_{p,\Omega} u_s = f_s \quad \text{in } \Omega$, with Dirichlet condition $u_s = g_s$ on $\partial\Omega$, where $f_s \nearrow f_1$ weakly in $L^p(\Omega)$ and $g_s \rightarrow g_1$ strongly as $s \rightarrow 1^-$. We prove that $\|u_s - u_1\|_{W^{s,p}(\Omega)} \rightarrow 0 \quad \text{as } s \rightarrow 1^-$. We reach this conclusion by relying on several key ingredients, including robust interpolation inequalities, robust Poincaré-type inequalities, convergence from nonlocal to local trace spaces, and Γ -convergence of the associated nonlocal nonlinear energy functionals to the corresponding local ones.

Hidde Schönberger (University of Twente)

Homogenization of nonlocal exchange energies in micromagnetics

We study the homogenization of nonlocal micromagnetic functionals incorporating both symmetric and antisymmetric exchange contributions under the physical constraint that the magnetization field takes values in the unit sphere. Assuming that the nonlocal interaction range and the scale of heterogeneities vanish simultaneously, we capture the asymptotic behavior of the nonlocal energies by identifying their Γ -limit, leading to an effective local functional expressed through a tangentially constrained nonlocal cell problem. Our proof builds upon a characterization of the two-scale limits of nonlocal difference quotients, yielding a nonlocal analog of the classical limit decomposition result for gradient fields. To deal with the manifold constraint of the magnetization, we additionally prove that the microscopic oscillations in the two-scale limit are constrained to lie in the tangent space of the sphere.

The talk is based on a joint work with Rossella Giorgio and Leon Happ.

Markus Gahn (University of Augsburg)

Derivation of effective interface laws for fluid flow through thin porous layers

We rigorously derive effective interface laws governing fluid flow through a thin elastic porous membrane separating two bulk fluid domains. The microscopic geometry consists of a periodically structured membrane composed of a solid and fluid-filled pores, with characteristic thickness and periodicity of order ε , small compared to the size of the bulk regions. The fluid flow is described by the stationary Stokes equations for incompressible fluids.

We consider both rigid and elastic solid. For the latter, we consider linear elasticity with a linearized fluid-structure interaction. Further, we deal with different scalings of the elastic stress tensor, leading to qualitatively different macroscopic behavior.

The derivation of the macroscopic model is based on compactness methods with respect to the two-scale convergence adapted to thin domains with oscillatory microstructure, where the membrane is asymptotically reduced to an effective interface. Across this interface, transmission conditions of Navier-slip type are derived, incorporating dynamic displacement effects in the case of an elastic solid.

Depending on the scaling for the elasticity tensor, the effective displacement satisfies either a membrane-type equation or a Kirchhoff-Love plate equation.

Dominik Engl (KU Eichstätt-Ingolstadt)

Derivation of variational membrane models in the context of anisotropic nonlocal hyperelasticity

Motivated by the analysis of thin structures, we study the variational dimension reduction of hyperelastic energies involving nonlocal gradients to an effective membrane model. When rescaling the thin domain, isotropic interaction ranges naturally become anisotropic, leading to the development of a theory for anisotropic nonlocal gradients with direction-dependent interaction ranges. Unlike existing nonlocal derivatives with finite horizon, which are defined via interaction kernels supported on balls of positive radius, our formulation is based on ellipsoidal interaction regions whose principal radii may vanish independently. This yields a unified framework that interpolates between fully nonlocal, partially nonlocal, and purely local models. Employing these tools, we present a Gamma-convergence analysis for the nonlocal thin-film energies. The limit functional retains the structural form of the classical membrane energy, and the classical local model is recovered precisely when all interaction radii vanish.

Lucas Fix (University of Augsburg)

Uniform Korn inequalities for variable coefficients and extension operators for perforated domains

We present generalisations of Korn's inequality for functions with vanishing mean rotation taking the form

$$\|u - P_F u\|_{W^{1,p}(\Omega)} \lesssim \|e_F(u)\|_{L^p(\Omega)} \quad \text{for all } u \in W^{1,p}(\Omega),$$

where $e_F(u) = \text{sym}(F^{-1}\nabla u)$, $F \in \mathcal{F}$, denotes a variable-coefficient linear strain operator for admissible families $\mathcal{F} \subset L^\infty(\Omega; \text{GL}(n))$, and P_F is a projector onto the kernel $\mathcal{N}(e_F)$. Particular emphasis is on estimates uniform in $F \in \mathcal{F}$, with a focus on families $\mathcal{F} \subset C^0(\Omega; \text{GL}(n))$ and families $\mathcal{F}$ consisting of gradients of Lipschitz deformations. Based on these results, we present an abstract framework to define F -strain preserving extension operators $E_F : W^{1,p}(Y_*)n \rightarrow W^{1,p}(Y)n$ resp. $E_{F,\varepsilon} : W^{1,p}(\Omega_\varepsilon)n \rightarrow W^{1,p}(\Omega)n$, the latter one by means of a local-to-global construction, for bounded Lipschitz domains $Y_* \subset Y \subset \mathbb{R}^n$ resp. periodically perforated domains $\Omega_\varepsilon \subset \Omega \subset \mathbb{R}^n$. These operators are shown to satisfy estimates with constants independent of $F \in \mathcal{F}$ and $\varepsilon > 0$, and also include the case of coefficients F_ε , $\varepsilon > 0$, that oscillate at the microscopic scale.

This is joint work with Markus Gahn and Malte A. Peter.

Joshua Kortum (JMU Würzburg)

Semigroup Estimates and Instability for the Linearized Prandtl Equations

We present semigroup estimates for the linearized Prandtl equations, which form a central component of a recent result on the ill-posedness of the Prandtl equations in Gevrey-4 spaces. More precisely, we derive semigroup bounds via energy estimates that feature Gevrey-2 regularity in the tangential variable and L^2 -integrability in the normal direction. Furthermore, we establish a connection between the linearized dynamics and the time evolution of the non-self-adjoint harmonic oscillator. Finally, we analyze a toy dynamical system to illustrate the complex behavior of semigroups generated by non-self-adjoint operators, and we utilize this model to explicitly distinguish Hadamard ill-posedness from weaker notions of ill-posedness present in the literature.

This is joint work with Francesco De Anna.

André Schlichting (Ulm University)

An entropic gradient-flow structure for the Dirichlet-to-Neumann evolution via boundary cell problems

This talk concerns the variational derivation of the Dirichlet-to-Neumann evolution on the boundary of a bounded domain, obtained as a multiscale limit of accelerated bulk diffusion with suitable mass exchange to the boundary. Each prelimit equation is a gradient flow of the relative entropy and passing to the limit yields a gradient structure characterized by a multidimensional cell problem that links boundary concentrations to boundary fluxes. This cell formula provides a multidimensional generalization of the one arising in diffusion-to-reaction or thin-membrane limits. The cell formula itself has an R-R* splitting, thereby exhibiting the Dirichlet-to-Neumann evolution as a generalized gradient flow.

Joint work with Georg Heinze and Mark Peletier.

Leonard Kreutz (TU Munich)

Γ -Expansion of the Cahn-Hilliard functional with Dirichlet boundary conditions

In this talk, we present the second-order asymptotic development of the Cahn-Hilliard functional under Dirichlet boundary conditions via Γ -convergence. We begin by reviewing results from the literature on the asymptotic expansion of the Cahn-Hilliard functional. Subsequently, we discuss our ongoing research focused on the Cahn-Hilliard functional with Dirichlet boundary conditions. In particular, we examine the case where no interior interfaces are present and highlight several open questions for future investigation. This seminar is based on work in collaboration with Irene Fonseca and Giovanni Leoni.

Noah Piemontese-Fischer (University of Bonn)

Scaling laws for incompatible two-well energies in geometrically linearized elasticity

We study a variational model of microstructures in shape-memory alloys, derived from the geometrically linearized theory of elasticity. Specifically, we consider a two-well energy with a singular perturbation term measuring the surface energy and seek to characterize the scaling behavior of optimal microstructures as the weight of the surface energy tends to zero. While the scaling behavior is well understood in the *compatible* setting, namely, when the stress-free strains are symmetrized rank-one connected and the prescribed linear boundary data are a convex combination of the stress-free strains, very little is known when the compatibility assumptions are violated.

In this talk, we discuss new results on scaling laws for the *incompatible* two-well problem, building on the work of Kohn [1], who studied the incompatible setting without surface energy. In particular, we prove matching upper and lower bounds. The upper bounds are established via branching constructions, while the lower bounds follow from a Fourier-based argument. The results are based on the article [2].

[1].Robert V. Kohn. The relaxation of a double-well energy. Contin. Mech. Thermodyn., 1991.

[2].Noah Piemontese-Fischer. The energy scaling behavior of a class of incompatible two-well problems. arXiv:2512.12014, 2025.

Timo Hofmann (University of Bonn)

Energy Scaling Laws for a Three-Well Singular Perturbation Problem

Models for shape-memory alloys based on minimizing a non-convex elastic energy are scale-invariant and thus cannot predict the characteristic length scale of the fine microstructures observed in nature. We consider a singular perturbation contribution in the form of a surface energy term, $\varepsilon |\nabla^2 u|(\Omega)$, which gives rise to a competition between the bulk and surface energies and determines an optimal microstructure length scale. This talk explores this model in a simplified, non-objective setting without gauges, focusing on a specific three-well target space $\mathscr{K}$. We establish matching upper and lower bounds. The presentation will briefly outline the upper bound construction, before focusing primarily on the ideas for the proof of the lower bound via discrete averaging techniques and localization methods. This talk is based on a joint project with L. Machill and A. Rüländ.

Timo Ziereis (TU Munich)

Emergence of Rigid Polycrystals from Ionic Particle Systems

We consider a discrete-to-continuum limit for ionic particle systems. At the surface scaling, we provide a full Γ -convergence result of the discrete system to an anisotropic perimeter-type functional, which models the behavior of crystalline boundaries in polycrystals. We focus on the discrete system to show that, depending on the orientation mismatch between crystals, different phenomena appear. Time permitting, we will discuss a formula for solid-vacuum grain boundaries and possible generalizations.

This talk is based on joint work with L. Kreutz (Technical University of Munich).

Felix Seifert (KU Eichstätt-Ingolstadt)

A general compactness result for localization of nonlocal gradients

Localization results provide a rigorous way to relate nonlocal models to their local counterparts. In the context of variational models, Γ -convergence together with the necessary compactness results offers the natural framework for analyzing such limit processes. In this talk, we discuss this approach for problems in which nonlocality arises through nonlocal gradients, as in models of nonlocal hyperelasticity. Whereas previous works often focused on kernels resulting from a specific construction, such as rescaling, or on special classes, like fractional kernels, we instead consider arbitrary sequences of radial kernels that satisfy suitable localization, monotonicity, and regularity conditions. This yields a more general approach that both encompasses various earlier results and extends them further, in particular by allowing for kernels with unbounded support and even non-integrable kernels. The main technical difficulty lies in the proof of compactness, which relies on establishing a nonlocal Poincaré inequality with uniform constants, as well as a uniform continuous embedding into a nonlocal Sobolev space given by a fractional reference kernel. To illustrate the scope of our results, we will show how different relevant examples fit into this setting.

This is joint work with Carolin Kreisbeck (KU Eichstätt-Ingolstadt) and Carlos Mora-Corral (Universidad Autónoma de Madrid).

Sonia Fliss (ENSTA Paris)

Scattering from a random thin coating of nanoparticles: the Dirichlet case

We consider in this talk the time-harmonic scattering by a heterogeneous object covered with a thin layer of randomly distributed nanoparticles. The size of the particles, their distance between each other and the layer's thickness are all of the same order but small compared to the wavelength of the incident wave. Solving the Helmholtz equation in this context can be very costly. Moreover, the simulation depends on the given distribution of particles, that is not known exactly in general (only the probability law is known). To circumvent this, we propose, via a multi-scale asymptotic expansion of the solution, an effective model where the layer of particles is replaced by an equivalent boundary condition. The coefficients that appear in this equivalent boundary condition depend on the solutions to corrector problems of Laplace type defined on unbounded random domains. Under the assumption that the particles are distributed given a stationary and mixing random point process, we establish quantitative error estimates for the effective model and present numerical simulations that illustrate our theoretical results.

David Wiedemann (TU Dortmund)

Effective Transport Induced by Time-Oscillating Microstructures

Time-periodic oscillations of a microstructure can induce transport effects at the macroscopic scale. We study this phenomenon for diffusion in a porous medium whose spatial and temporal microscopic scales are both of order ϵ . Although the microscopic equation is purely diffusive, homogenisation yields an effective advection--diffusion equation. Using the two-scale transformation method, we characterise the effective coefficients including the effective advective transport velocity through cell problems. A macroscopic modulation of the pulsations leads to a space- and time-dependent advection velocity and can generate arbitrary locally varying velocity fields. Finally, we present preliminary considerations for Stokes flow in pulsating pore geometries and discuss how periodic microscopic deformations may generate effective macroscopic fluid transport.

Wolfgang Reichel (KIT)

Optimal data-driven solutions for a population growth model

We are considering stationary solutions of a population growth model with diffusion. Conventional models, like the Verhulst model of logistic growth, assume exact knowledge of the nonlinear dependance $(x, u) \mapsto r(x, u)$ where u is population size, x position in $\Omega \subset \mathbb{R}^n$ and $r(x, u)$ is the net per-capita reproduction rate of the population. In our model we replace the reproduction law $(x, u) \mapsto r(x, u)$ by a data-set $D = \{(x_\beta, u_\beta, r_\beta) : \beta \in B\} \subset \mathbb{R}^{n+2}$ and look for function pairs $(u, r) \in H^1(\Omega) \times L^\infty(\Omega)$ which satisfy

$$-\Delta u = ru \text{ in } \Omega, \quad \frac{\partial u}{\partial \nu} = 0 \text{ on } \partial\Omega$$

and which match the data in an optimal way (optimal data-driven solutions). We answer three questions: existence, consistency and convergence. Consistency addresses the question what the relation between a classical solution and an optimal data driven solution is, if the data set is generated from a known reproduction law $(x, u) \mapsto r(x, u)$. Convergence addresses the question to what limit optimal data-driven solutions converge if the data sets converge.

The results rely on a variational characterization of optimal data-driven solutions, Γ -convergence methods and aspects of measure theory.

The talk is based on joint work with Laura Baldelli (KIT) and Paolo Malanchini (Università degli Studi di Milano - Bicocca), cf. arxiv:2608.01242.

Nicola Zamponi (University of Augsburg)

Analysis of a phase-field model for organic solar cells

We consider a model describing the formation of solar cells based upon an organic semiconductor. Such cells arise from the phase separation of an electron-donor (polymer) and an electron-acceptor (non-fullerene acceptor) catalyzed by solvents evaporation. This process is described by a phase-field model with four different phases (polymer, NFA, solvent, air) and an order parameter denoting the fraction of the mixture in vapor phase, resulting in a coupled system of incompressible Navier-Stokes, multispecies Cahn-Hilliard and Allen-Cahn equations with cross-diffusion terms depending on the gradient of chemical potentials. We prove the existence of global-in-time weak solutions to the system without size constraints on the data. The solutions dissipate the total energy and involve an ad-hoc variational formulation of the definition of the diffusion fluxes, which is made necessary by the degeneracy of the mobility matrix. The proof is based on a-priori estimates obtained from the energy balance and approximated equations. A crucial bound for the second derivatives of the phases is obtained from a careful estimate of the terms appearing in the energy balance involving the chemical potentials, based on suitable structural assumptions on the degenerate mobility matrix. The approximated scheme relies on an energy-preserving time discretization and ad-hoc regularizing terms. While the existence of approximated solutions follows from a fix point argument, compactness of the sequence of approximating solutions is obtained from Aubin-Lions Lemma.

Annika Bach (Eindhoven University of Technology)

The antiferromagnetic XY-model on the layered triangular lattice: Chirality transitions in a rigid regime

The antiferromagnetic XY -model on the (layered) triangular lattice is a canonical example of a geometrically frustrated spin system. In general, the XY -model energy is defined on discrete order parameters u mapping from the lattice to the unit circle S^1 and in the antiferromagnetic model this energy favors u to take opposite values in neighboring lattice points. However, the structure of the (layered) triangular lattice does not allow u to do this for all nearest neighbors simultaneously. In other words, the energy cannot be minimized by minimizing all pairwise interactions. This is what is referred to as geometric frustration. As a result of frustration, the energy landscape of these systems becomes very rich, meaning that energy can concentrate on many different scales. In this context, Γ -convergence is a useful tool to pass to a (simpler) continuum model while still keeping the scale-dependent relevant information. In this talk, we will carry out such a discrete-to-continuum Γ -convergence analysis in a scaling regime where in the continuum limit the so-called chirality variable develops sharp interfaces.

This is joint work with Rafael Galleze (TU Eindhoven).

Francesco De Anna (JMU Würzburg)

Hadamard ill-posedness of the linearised Prandtl equations in several spaces of high tangential regularity

The Prandtl equations model viscous boundary layers near solid surfaces at high Reynolds numbers. Their well- and ill-posedness are central mathematical questions, closely tied to the physics of boundary layer separation. It has long been established that these equations linearised around suitable non-monotone shear flows exhibit strong instabilities at high tangential frequencies. Beginning with

the work of Gérard-Varet and Dormy (2010), these instabilities have characterised the failure of uniform Sobolev bounds for the solution operators.

This talk follows the presentation by my co-author, Dr Joshua Kortum, who explained why these earlier results do not by themselves establish Hadamard ill-posedness for more regular initial data, such as Gevrey data, prescribed at a fixed initial time.

Gevrey spaces are particularly relevant, as they provide a natural framework for analysing the high-frequency amplification observed in numerical simulations and physical experiments.

We present a recent result establishing Hadamard ill-posedness in tangential Gevrey class 4 for the equations linearised around suitable smooth, time-dependent shear flows solving the unforced nonlinear Prandtl equations.

More precisely, we construct smooth, compactly supported initial data of Gevrey class 4 in the tangential variable for which the linearised system admits no weak solution in the natural energy class on any positive time interval.

The proof combines an inner-outer gluing construction with a perturbative argument. It builds on a technique developed in earlier joint work with Kortum that provides an explicit characterisation of the unstable eigenmodes for quadratic shear flows.

Alex Suri (University of Paderborn)

Geodesic interpretation of the global quasi-geostrophic equations

We present an interpretation of the global shallow water quasi-geostrophic equations on the sphere $\mathbb{S}^2$ as a geodesic equation on the central extension of the quantomorphism group on $\mathbb{S}^3$. The study includes deriving the model as a geodesic equation for a weak Riemannian metric, demonstrating smooth dependence on the initial data, and establishing global-in-time existence and uniqueness of solutions.

We further show that the Lamb parameter in the model has a stabilizing effect on the dynamics: if it is sufficiently large, the sectional curvature along the trade-wind current is positive, implying the existence of conjugate points.

Leonardo Del Grande (University of Twente)

Multi-Headed Transformer Architectures as Time-dependent Wasserstein Gradient Flows

Transformer architectures have revolutionized language processing, yet existing mathematical models rely on strong simplifying assumptions, notably single-head and depth-independent weights. We narrow this gap by modelling multi-headed transformers as time-dependent Wasserstein gradient flows of an interaction energy capturing the attention mechanism. The explicit time dependence accommodates distinct weights for each head and layer, with no constraints on initialization. We prove that, under a suitable integrability assumption on the weight evolution, every element of the ω -limit set is a stationary point of the energy at a limiting weight distribution. We also establish robustness under input perturbations, uniqueness of the flows, and Γ -convergence of the perturbed energies, yielding convergence of the corresponding gradient flows. Numerical experiments confirm the energy-dissipation identity and clarify the asymptotic behavior in Ornstein–Uhlenbeck and oscillating-weight regimes.

Support

The organizers acknowledge and are thankful for the partial support by CAAPS Augsburg.

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